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Multifactor Trend Model of Sustainable Company Growth in the Context of Competition and Inflation

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Abstract

The present research analyzes corporate growth against the background of competition and inflation, with a focus on the sustainable growth concept. The existing sustainable growth models are more suitable for solving the inverse problem when funding sources are defined at a predetermined growth rate, and less effective for solving the direct problem of sales volume planning when initial data is known. This is due to the fact that traditional models leave out external growth factors. Although inflation models of sustainable growth have remedied the situation, they are still dismissive of competition. The offered multifactor trend model of sustainable growth bridges this gap by taking into consideration the key growth drivers: investment ratio, asset turnover, return on sales, financial leverage and dynamics of product and resource prices. Differential and integral calculus methods were applied to develop the model, and financial ratios are considered as dynamic values described by the trends that are indicative of the external environment impact. The model potential is exemplified in various industries. It provides an opportunity to model the company's entire life cycle, including a period of decline, and may be used as a strategic planning tool. The model describes natural, typical and logistics growth and may be used at the early stages of the life cycle when data is limited.

Keywords: inflation, financial ratio, trend, competition, sustainable growth, logistics growth, industry growth, life cycle, current and strategic planning

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To my granddaughter Valentina!

Introduction

A multivarious mathematical apparatus, in particular differential calculus and regression analysis [1], is used to study companies' performance. The importance of regression models [2–10] resides in the opportunity to take into consideration non-financial growth factors, while differential models are constructed exclusively on the basis of accounting data. An example of the latter are sustainable growth models [11–16], which are differential equations with the coefficients that show the relationship between the balance sheet and revenue. While econometric studies confirm statistical hypotheses laid down descriptively on the basis of general theoretical premises, the models that evaluate growth in reliance on a cause-and-effect relationship pertain to fundamental analysis. Initially, such models took into consideration only the factors that are indicative of internal capabilities of the company, and subsequently – external ones related to inflation [17]. The inflation-related components of the product and manufacturing resource prices may be taken into account separately [18]. The full potential of sustainable growth models is not brought out, especially taking into account the fact that they take inflation into account.

Integration of differential models allows us to analyze the long-range trends of revenue time series. When integrating the sustainable growth equations, exponential trends with a constant increment rate are obtained. To depart from a simple exponent, it is incumbent to take into consideration the temporal dynamics of the financial ratios involved in the integration. The paper dedicated to the forecasting of financial ratios used to define the company value provides the necessary information [19]. This research covers the ratios that constitute a decomposition of return on equity according to the DuPont chart. It was established that as time goes by, they approximate the typical levels which, according to the industry-adjusted DuPont model [20], may be represented by industry-specific medians. These dynamics may be caused by competition, which eliminates the deviation of the ratios from typical levels. There is a belief that this is the way to ensure the optimal operational structure of a company. Since profitability directly influences the sustainable growth rate, the increase in complexity of the models usually consists in adding of new indicators from the DuPont chart. As a result, information on long-term trends in the majority of financial ratios significant for company growth has been accumulated by now.

It follows from the industry-adjusted DuPont model that over time the increment rate approximates the industry level. This assumption is in line with the two-stage business value estimation model which divides the time horizon into the forecast and post-forecast periods. The initial stage may be characterized by an increased increment rate because the company has a “window of opportunity” due to competitive advantages [21]. As and when they are lost, the stable growth stage starts. During this stage, the window of opportunity grows simultaneously with the in-

dustry. This stage may be described by an exponent with a constant increment rate. The transition from one stage to another is accompanied by changes in the value of financial ratios that influence the rate of revenue increment: return on sales, financial leverage, undistributed profit, asset turnover. Empirical data [19; 20; 22] shows that there may be different directions of financial ratios' deviation from typical values. Therefore, at the initial stage, apart from the “window of opportunity”, a reverse situation may occur when over time the company improves its performance achieving the industry average values.

If we come to the macroeconomic level, according to the Harrod-Domar model [23, p. 64], the exponential growth with a constant increment rate called the guaranteed [24] or equilibrium [25] one is possible in the circumstances of sustainable development when the rates of increment in investment and production output are identical and equal to the product of the savings rate and the marginal productivity of capital. Premised on the fact that with time business growth approximates the industry growth, which, in turn, depends on the macroeconomic environment, it is fair to assume that there is a relationship between the company's ratios and macroeconomic indicators that determine the equilibrium growth. The sustainable growth and Harrod-Domar economic growth models have something in common. In both cases a fixed share of income goes back to the production process, thus, increasing the used funds and providing the basis for exponential growth. This is indicative of the prospect of applying sustainable growth models at higher levels of economic indicators' aggregation.

The purpose of this research is to develop a sustainable growth model that takes into consideration the impact of inflation and competition. For this purpose, trends of financial ratios are added to the inflation model with a transfer from the differential to the integrated form. The trends of inflation of manufacturing resource price and the spread of inflation of company's product and manufacturing resource price take price environment changes into consideration. Trends of the ratios of undistributed profit, asset turnover, return on sales and financial leverage manifest changes in the operating structure of the company under the pressure of competition.

Model

Two similar indicators are used to analyze the economic value dynamics: growth rate and increment rate. They describe the dynamics using the relationship between the values that pertain to different time points. For revenue that takes on the value of S_t at time t , the periodic growth and increment rates are described by the expressions

$\frac{S_{t+T}}{S_t}$ and $\frac{\Delta S_{t+T}}{S_t}$, where T is duration of the analyzed period; $\Delta S_{t+T} = S_{t+T} - S_t$ is the absolute increment.

The actual values of periodic indicators may be calculated on the basis of accounting data. With $T \rightarrow 0$ we get the revenue increment rate of $\frac{dS_t}{dt} \frac{1}{S_t}$ where $\frac{dS_t}{dt}$

is the rate of revenue receipt. The relationship between the increment rate and the periodic growth rate is as follows:

$$\int_t^{t+T} \left(\frac{dS_{t'}}{S_{t'}} \frac{1}{dt'} \right) dt' = \int_t^{t+T} \frac{dS_{t'}}{S_{t'}} = \ln \left(\frac{S_{t+T}}{S_t} \right).$$

Some sustainable growth models (for example, [13]) describe the periodic increment rate, while others (for example, [18]) describe the increment rate. The present paper analyzes the revenue increment rate.

Fundamental Equation

The revenue increment rate $g_t = \frac{dS_t}{dt} \frac{1}{S_t}$ of a steadily

growing business is proportional to the equity increment rate $\frac{dE_t}{dt} \frac{1}{E_t}$:

$$g_t = \left(\frac{dE_t}{dt} - \frac{dP_t}{dt} + \frac{dB_t}{dt} \right) \frac{1}{E_t} = b_t \frac{dE_t}{dt} \frac{1}{E_t}, \quad (1)$$

where $\frac{dE_t}{dt}$ is the rate of change in equity E_t before making payments to shareholders; $\frac{dP_t}{dt}$ and $\frac{dB_t}{dt}$ are the rates

of dividend payment and stock issue;

$$b_t = \frac{\left(\frac{dE_t}{dt} - \frac{dP_t}{dt} + \frac{dB_t}{dt} \right)}{\left(\frac{dE_t}{dt} \right)}$$

is the investment ratio responsible for payments to shareholders. If only net income was the source of corporate equity growth, equity equals the undistributed net profit ratio.

According to the inflation model [18] the rate of equity change equals:

$$\begin{aligned} \frac{dE_t}{dt} = & m_t S_t + (i_t - j_t)(1 - \phi_t) S_t - \\ & - e_t \alpha_t (1 - \phi_t) [\beta_{1,t} (C_t + I_t - L_t) + \beta_{2,t} F_t] S_t + \\ & + j_t F_t (\kappa_t + o_t) (1 - \phi_t) S_t + \\ & + j_t [I_t + F_t - F_t (\kappa_t + o_t)] S_t, \end{aligned} \quad (2)$$

where m_t is the actual return on sales, which equals the ratio of net profit exclusive of inflation to annual revenue S_t ; C_t is the ratio of cash and net receivables to annual revenue; L_t is the ratio of accounts payable and other spontaneous liabilities to annual revenue; I_t is the ratio of inventories to annual revenue; F_t is the ratio of net fixed assets to annual revenue; κ_t is the depreciation rate; o_t is the rate of unfore-

seen depreciation, $\beta_{1,t} = \frac{D_{1,t}}{(C_t + I_t - L_t) S_t}$ is the share of

working capital financed from debt $D_{1,t}$; ϕ_t is the profit tax rate; $\beta_{2,t} = \frac{D_{2,t}}{F_t S_t}$ is the share of fixed assets financed from

debt $D_{2,t}$; α_t is the share of debt with a free-floating interest rate; i_t is inflation of the corporate product prices; j_t is inflation of manufacturing resource prices; e_t is the adjustment of the loaned funds' interest rate due to inflation. When the price dynamics are described, inflation serves as an analogue of the increment rate.

It follows from the definitions of $\beta_{1,t}$ and $\beta_{2,t}$ that financial leverage $(D/E)_t$ equals

$$\left(\frac{D}{E} \right)_t = \frac{\beta_{1,t} (C_t + I_t - L_t) S_t + \beta_{2,t} F_t S_t}{(1 - \beta_{1,t}) (C_t + I_t - L_t) S_t + (1 - \beta_{2,t}) F_t S_t}, \quad (3)$$

where debt D_t and equity E_t are described by the following expressions: $D_t = D_{1,t} + D_{2,t} = \beta_{1,t} (C_t + I_t - L_t) S_t + \beta_{2,t} F_t S_t$ and $E_t = (1 - \beta_{1,t}) (C_t + I_t - L_t) S_t + (1 - \beta_{2,t}) F_t S_t$. Henceforward, fractions in parentheses are considered as financial ratios.

Model (1) takes into consideration the possibility of raising equity by means of stock issue ($b_t > 1$). Apart from that, undistributed net profit and asset revaluation contribute to equity growth. According to (2):

a) Net income exclusive of inflation $m_t S_t$:

grows by $(i_t - j_t)(1 - \phi_t) S_t$ due to the difference in inflation of product and manufacturing resource inflation;

decreases by $e_t \alpha_t (1 - \phi_t) [\beta_{1,t} (C_t + I_t - L_t) + \beta_{2,t} F_t] S_t$ because the interest on borrowed funds is paid;

rises by $j_t F_t (\kappa_t + o_t) (1 - \phi_t) S_t$ because depreciation does

not take into consideration the increase in the cost of fixed assets caused by inflation;

b) the final growth factor is related to an increase in the cost of previously purchased inventories and fixed assets by $j_t [I_t + F_t - F_t (\kappa_t + o_t)] S_t$, and it is shown in (2) as an

increment in add-on capital due to increases in asset value. Asset turnover financed without accounts payable and other

spontaneous liabilities $\left(\frac{S}{A} \right)_t$ equals

$$\left(\frac{S}{A} \right)_t = (C_t + I_t - L_t + F_t)^{-1}, \quad (4)$$

and on this basis, we obtain the fundamental equation of the trend model of sustainable growth taking into consideration (1)–(3):

$$g_t = b_t \left\{ \left[\left(\frac{S}{A} \right)_t (m_t + z_t(1-\phi)) + j_t \varepsilon \right] \left[1 + \left(\frac{D}{E} \right)_t \right] - e_t \alpha (1-\phi) \left(\frac{D}{E} \right)_t \right\}, \quad (5)$$

where $z_t = i_t - j_t$ is the spread between inflation of product and manufacturing resource prices;

$\varepsilon = \frac{I_t + F_t [1 - \phi(\kappa_t + o_t)]}{C_t + I_t - L_t + F_t}$ is the ratio of revaluated assets

to the assets financed exclusive of spontaneous liabilities. When integrating this indicator, the share of debt with the free-floating interest rate and income tax are considered to be constant, therefore in (5) they are presented without the time index. The rest of the ratios are considered as dynamic quantities.

Trend Model

To take into account a change in financial ratios over time, we introduce their long-term trends into equation (5). According to empirical data, they comply with the pattern represented by a modified exponent (see Appendix 1):

$$\begin{aligned} b_t &= h_0 + h_1 e^{\delta t}, & \left(\frac{S}{A} \right)_t &= f_0 + f_1 e^{\xi t}, \\ m_t &= c_0 + c_1 e^{\gamma t}, & \left(\frac{D}{E} \right)_t &= l_0 + l_1 e^{\lambda t}, \\ z_t &= y_0 + y_1 e^{\eta t}, & j_t &= n_0 + n_1 e^{\pi t}, \end{aligned} \quad (6)$$

with the following constant ratios:

$$\begin{aligned} p_0 &= h_0 f_0 c_0 (1 + l_0) + h_0 f_0 y_0 (1 - \phi) (1 + l_0) + h_0 n_0 \varepsilon (1 + l_0) - n_0 \alpha (1 + l_0) h_0 l_0, \\ p_1 &= [h_1 f_0 c_0 + h_1 f_0 y_0 (1 - \phi) + h_1 n_0 \varepsilon] (1 + l_0) - n_0 \alpha (1 - \phi) h_1 l_0, & q_1 &= \delta, \\ p_2 &= h_0 f_0 c_1 (1 + l_0), & q_2 &= \gamma, & p_3 &= h_0 f_0 y_1 (1 - \phi) (1 + l_0), & q_3 &= \eta, \\ p_4 &= h_0 n_1 [\varepsilon (1 + l_0) - \alpha (1 - \phi) l_0], & q_4 &= \pi, & p_5 &= h_0 f_1 (1 + l_0) [c_0 + y_0 (1 - \phi)], & q_5 &= \xi, \\ p_6 &= h_0 l_1 [f_0 c_0 + f_0 y_0 (1 - \phi) + n_0 \varepsilon - n_0 \alpha (1 - \phi)], & q_6 &= \lambda, \\ p_7 &= h_1 f_0 c_1 (1 + l_0), & q_7 &= \gamma + \delta, & p_8 &= h_0 f_0 c_1 l_1, & q_8 &= \gamma + \lambda, \\ p_9 &= h_1 l_1 [f_0 c_0 + f_0 y_0 (1 - \phi) + n_0 \varepsilon - n_0 \alpha (1 - \phi)], & q_9 &= \lambda + \delta, \\ p_{10} &= h_0 f_0 y_1 (1 - \phi) l_1, & q_{10} &= \eta + \lambda, & p_{11} &= h_0 n_1 l_1 \varepsilon - n_1 \alpha (1 - \phi) h_0 l_1, & q_{11} &= \pi + \lambda, \\ p_{12} &= h_0 f_1 c_1 (1 + l_0), & q_{12} &= \gamma + \xi, & p_{13} &= h_1 f_1 (1 + l_0) [c_0 + y_0 (1 - \phi)], & q_{13} &= \xi + \delta, \\ p_{14} &= h_0 f_1 y_1 (1 - \phi) (1 + l_0), & q_{14} &= \eta + \xi, & p_{15} &= h_0 f_1 l_1 [c_0 + y_0 (1 - \phi)], & q_{15} &= \xi + \lambda, \\ p_{16} &= h_1 f_0 y_1 (1 - \phi) (1 + l_0), & q_{16} &= \eta + \delta, & p_{17} &= h_1 n_1 \varepsilon (1 + l_0) - n_1 \alpha (1 - \phi) h_1 l_0, & q_{17} &= \pi + \delta, \\ p_{18} &= h_1 f_0 c_1 l_1, & q_{18} &= \gamma + \lambda + \delta, & p_{19} &= h_1 f_1 c_1 (1 + l_0), & q_{19} &= \gamma + \xi + \delta, \\ p_{20} &= h_1 f_1 l_1 [c_0 + y_0 (1 - \phi)], & q_{20} &= \xi + \lambda + \delta, & p_{21} &= h_0 f_1 y_1 (1 - \phi) l_1, & q_{21} &= \eta + \xi + \lambda, \\ p_{22} &= h_0 f_1 c_1 l_1, & q_{22} &= \gamma + \xi + \lambda, & p_{23} &= h_1 n_1 \varepsilon l_1 - n_1 \alpha (1 - \phi) h_1 l_1, & q_{23} &= \pi + \lambda + \delta, \end{aligned} \quad (8)$$

where $h_0, f_0, c_0, l_0, y_0, n_0$ are the final values of trends, which show their limit at $t \rightarrow \infty$ with deviations at the initial time point, which equal $h_1, f_1, c_1, l_1, y_1, n_1$ and the deviations' increment rates of $\delta, \xi, \gamma, \lambda, \eta, \pi$. To avoid trends' (6) tending to infinity, the deviations' increment rates should be less than zero.

The final values of trends may equal the typical values which, according to the DuPont industry-adjusted model, are equivalent to industry-specific medians. We are going to call a company with such final values typical. It is presumed that far from all companies are considered typical.

Corporate expenses are formed by expenses that are heterogeneous in terms of economic content and purpose, therefore, inflation of manufacturing resource prices is roughly equivalent to general economic inflation. According to Fisher's effect, it also determines the adjustment of the loaned funds' interest rate. Thus, these growth factors match in terms of value: $j = e$.

We subsequently obtain the differential multifactor trend model, which describes the revenue increment rate from equation (5), taking into consideration (6) and the equality $j = e$ (see Appendix 2):

$$g_t = p_0 + \sum_{k=1}^{27} p_k e^{q_k t}, \quad (7)$$

$$\begin{aligned}
 p_{24} &= h_1 f_1 y_1 (1 - \phi)(1 + l_0), & q_{24} &= \eta + \zeta + \delta, & p_{25} &= h_1 f_0 y_1 (1 - \phi) l_1, & q_{25} &= \eta + \lambda + \delta, \\
 p_{26} &= h_1 f_1 c_1 l_1, & q_{26} &= \gamma + \zeta + \lambda + \delta, & p_{27} &= h_1 f_1 y_1 (1 - \phi) l_1, & q_{27} &= \eta + \zeta + \lambda + \delta.
 \end{aligned}$$

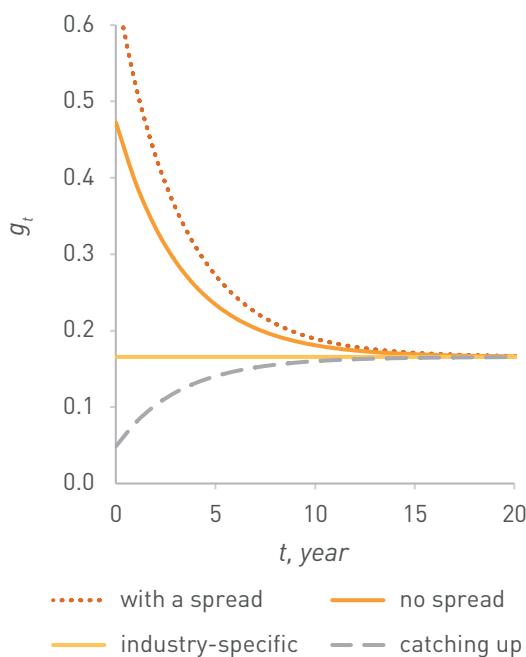
Having integrated (7), we obtain a multifactor trend model that describes the trajectory of the sustainable growth of company revenue:

$$S_t = S_0 \exp \left[p_0 t - \sum_{k=1}^{27} \frac{p_k}{q_k} (1 - e^{q_k t}) \right] \quad (9)$$

where S_0 is revenue at the initial time point.

Graphic Analysis

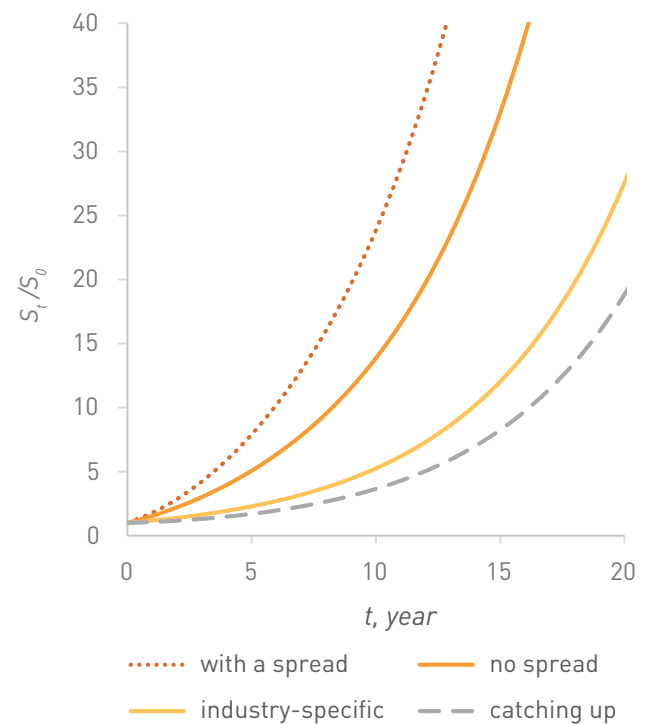
Figure 1. Increment rates



We are going to graphically analyze the differential (7) and integrated (9) models (Figures 1 and 2). The financial ratios applied to construct the graphs are presented in Appendix 3 (with the information used to build the graphs presented in Figures 7 and 8). The final values of the financial ratios' trends in all examples equal the industry values, therefore, as time passes, revenue increment rates become identical, approximating the industry level. The speed of achieving this state is defined by the deviation increment rates. They are the same for all financial ratios. The revenue increment rate depends on the spread of product and manufacturing resource prices. A positive spread accelerates sustainable growth significantly. A company that is catching-up is distinct from others by the negative price spread and decreased initial values of return on sales and investment ratio. When the industry growth stage is reached, the increment rate graph becomes straight. At this stage, the growth trajectory is described with a simple exponent. In

all other cases, despite the exponential form, the revenue growth is not simple because the increment rates are not constant. The presented dependencies plotted using the trends whose final values are equal to the typical ones, may be called typical. Atypical sustainable growth trajectories are considered below.

Figure 2. Sustainable growth trajectories



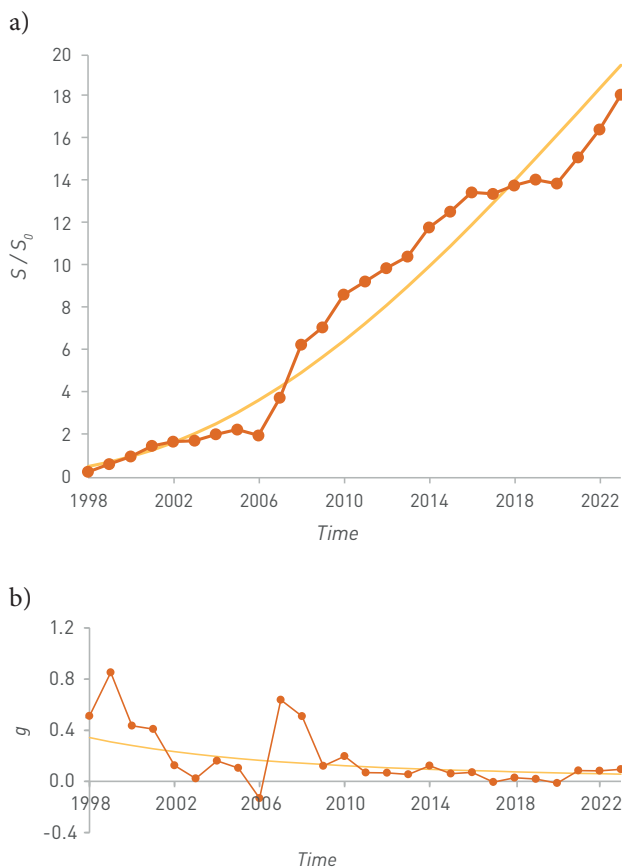
Data

The trend model takes into consideration the dynamics of financial ratios, providing an opportunity for us to study growth at a long-term time horizon. We apply it to calculate the growth trajectories (Figures 3a–6a) and revenue increment rates (Figures 3b–6b) of the companies operating in various fields. Theoretical graphs are indicated as solid lines while empirical graphs – as lines with markers. The accounting information is obtained in the SCRIN database. Inflation of product and manufacturing resource prices of companies was taken to be equal to general economic inflation, except for PJSC VimpelCom. For this company, product price dynamics was estimated by the average price per minute (APPM)¹. Based on the bank's right to determine the cost of lending unilaterally, the interest rate of all loans was considered to be free floating. Financial ratios' trends, except for the investment ratio, are constructed by approximating empirical data with equations (6). As at the

¹ The data from [26] was used.

end of the 1990s, accounting information was sometimes incomplete, therefore, the approximation procedure covered the period which started in the 2000s. The investment ratio depends not just on payments to shareholders, but also on changes in the asset value caused by inflation. It was calculated on the basis of the accounting data and turned out to be inaccurate because the analyzed companies extremely rarely revalued non-current assets. In order to bypass this obstacle, the model was calibrated using this ratio, while its trends remained within the statistical scatter of the empirical data. Comparison of the graphs shows that the modeling results are in line with the practice, except for the economic shock periods which disrupt the existing trends for a short time. This may be exemplified by perfor-

Figure 3. JSC Wimm Bill Dann



mance degradation of PJSC Aeroflot in 2020 as a result of a decline in air transportation caused by the COVID-19 pandemic (Figure 5).

Discussion

Up to the present time, theoretical analysis revolved around typical scenarios of company operations, which imply that over time financial ratios gravitate towards the levels preset by industry-specific medians. Empirical data analysis shows that the typical scenario is not always implemented. We are going to consider special cases of the trend model that demonstrate sustainable growth in other economic environments, which are described by theories and occur in practice.

Figure 4. PJSC VimpelCom

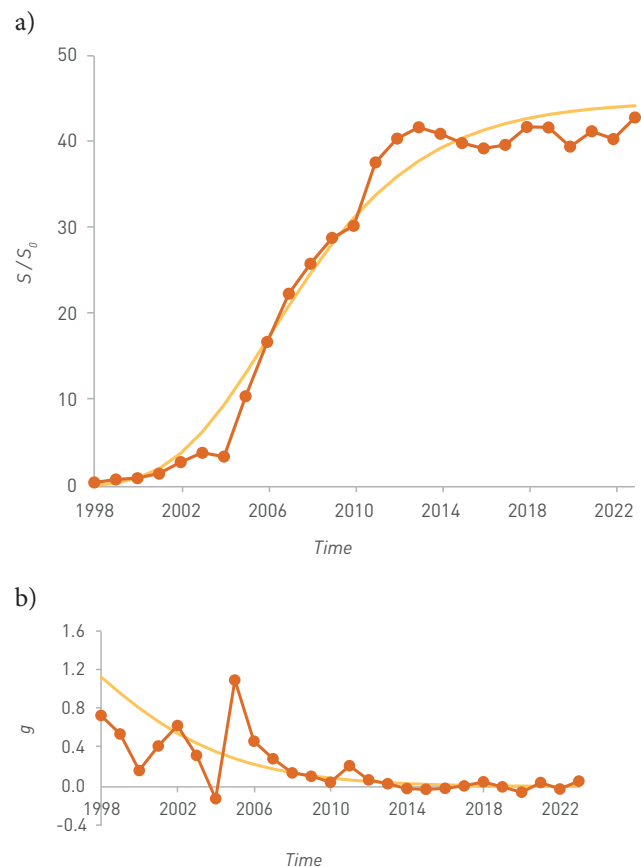
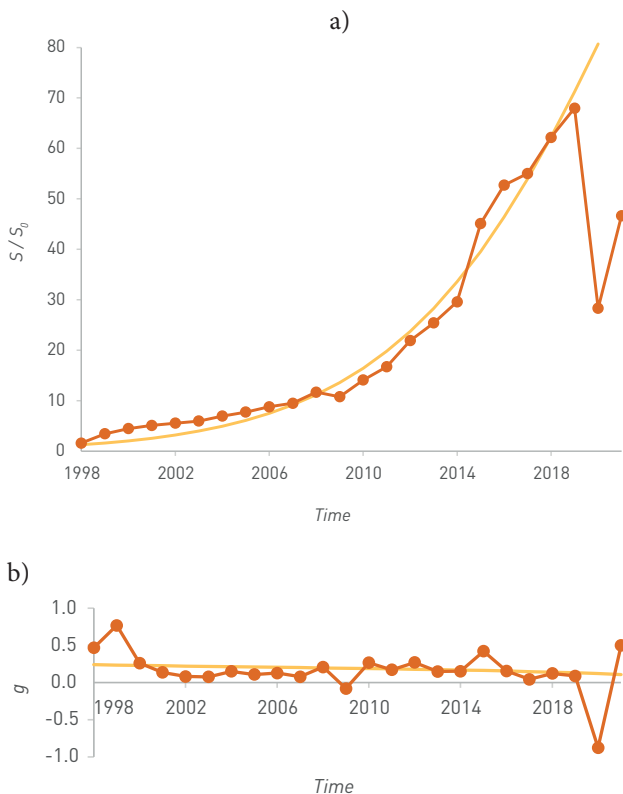
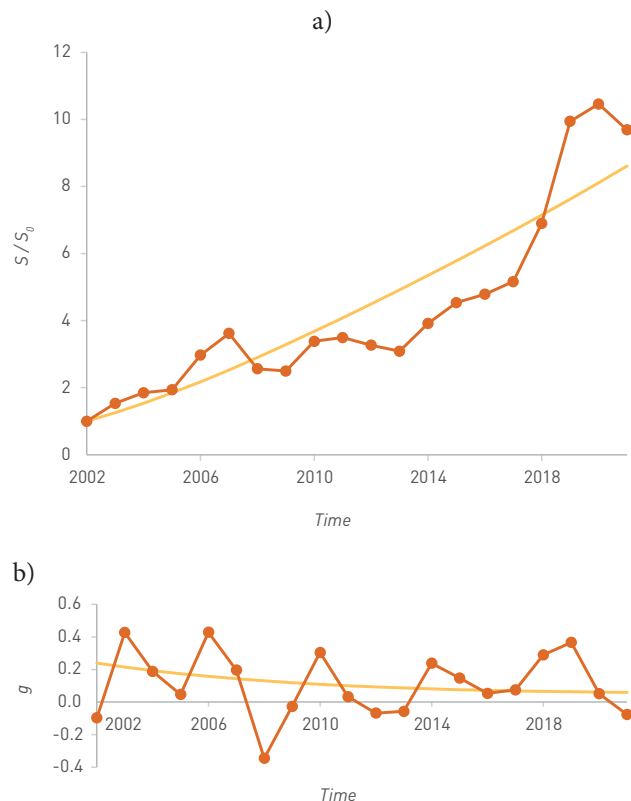


Figure 5. PJSC Aeroflot**Figure 6.** PJSC Mining and Metallurgical Company Norilsk Nickel

Natural Growth

Suppose that financial ratios remain unchanged over time. In order to model this situation, we zero out the increment rates of trend deviations (6) in the differential equation (7). After the integration of the obtained equation, we obtain the following model:

$$S_t = S_0 e^{\sum_{k=0}^{27} p_k t}, \quad (10)$$

that describes exponential growth with a constant increment rate. It may be observed in a competition-free environment when a company sells the manufactured products unhindered and allocates a part of income to production expansion. The typical levels are concealed here. If we consider the absence of competition to be the main feature of natural growth, equation (10) is its model. In practice the conditions required for natural growth emerge when a company stands at the origins of a new industry, defining its standards. Innovator enterprises called “gazelle companies” show aggressive growth well-described by a simple exponent [27]. Note that the time horizon covered by the model (10) is not limited to the initial time interval. The passage to the limit $t \rightarrow 0$ also provides an exponential dependence with a constant increment rate; however, this solution does not implicate an absence of competition.

Industry Growth

The industry-adjusted DuPont model shows that the typical levels of financial ratios are presented by industry-specific medians. So, having zeroed out the trend deviations

(6) from typical levels ($l_1 = h_1 = c_1 = f_1 = y_1 = n_1 = 0$) we obtain the following model instead of (9):

$$S_t = S_0 e^{p_0 t}, \quad (11)$$

where p_0 is the industry increment rate (annual). It is a time-constant value. At the macroeconomic level, equilibrium (guaranteed) Harrod-Domar growth is characterized by a constant increment rate. Hence, we get a “macroeconomic” method of p_0 calculation by means of the product of the industry-specific savings rate and the marginal productivity of capital. If we apply this method to calculate p_0 , then, based on the obtained value as a selection criterion we may find the companies close to the stage of industry growth and use them to determine the typical values of financial ratios without analyzing their time series. It should be noted that in practice it is impossible to achieve the stage of industry growth. This virtual state exists only on paper because the typical levels are attained by financial ratios only with $t \rightarrow \infty$.

"Logistics" Growth

The distinctive feature of exponential models is the fact that they cause unrestricted growth. It is absent in the models based on the diffusion equation where a logistic curve replaces the exponent [28; 29]. Some diffusion models were used successfully to describe the economic processes related to the diffusion of innovation and sale of high-tech products [30–33]. At the initial stage, the logistic dynamics are close to exponential dynamics and describe growth that is similar to natural one. The competitive pressure related

to innovators and imitators is still not strong at this stage. The curve smooths out gradually and reaches a plateau. Using the trend model (9), one may model sustainable growth similar to logistics growth - initially exponential, subsequently halting growth (Figure 7). It is obtained when the final trend values of certain financial ratios are set to zero. One may see the combinations necessary for this purpose in equation (5). The figure shows the version with the zero final values of return on sales, interest rate spread, manufacturing resource price inflation and financial leverage. A similar result is obtained when only the final value of the investment ratio is set to zero.

Figure 7. "Logistic" trajectory of sustainable development

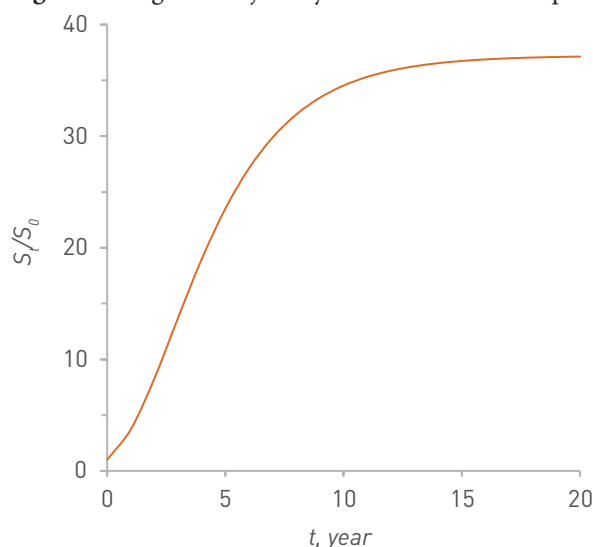
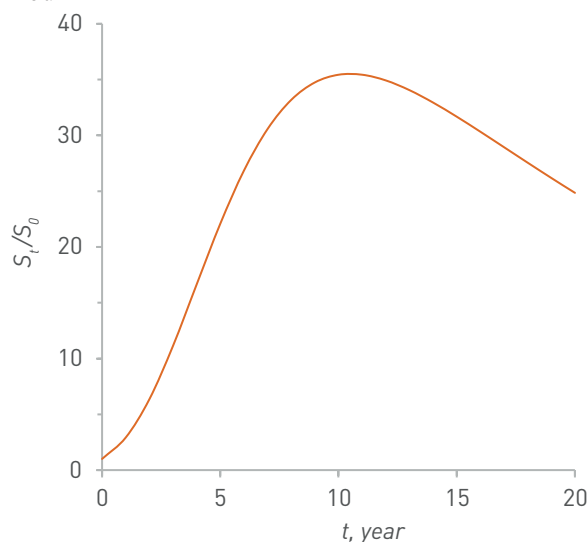


Figure 8. Sustainable growth trajectory with a decline period



Life Cycle

According to the life cycle concept, the growth period within which financial ratios approach the typical levels is not necessarily the last one [34–37]. At any stage, a failed

company may go bankrupt having passed the zero growth and decline periods. The typical financial ratio trends neutralize such cases because they are built on the basis of the industry-specific medians. In order to model the history of a failed company by applying (7) and (9), it is necessary to use the individual final trend values, which lead to a negative revenue growth rate, instead of typical ones (Figure 8). Different combinations are possible. All prosperous companies are alike, while each failed company goes bankrupt in its own way². The abovementioned curve is obtained at the negative final return on sales. It covers the zero growth and decline stages.

Conclusion

While previous sustainable growth models were effective in solving the inverse problem when funding sources were defined at a predetermined increment rate, inflation-related modifications may be applied to solve the direct problem, namely, planning revenue at predetermined values of financial ratios. The trend model presented in this paper is best suited for this purpose because it eliminates the most significant drawback of sustainable growth models, i.e., overlooking of external environment conditions. The typical trends of financial ratios allow to apply the model at early stages of the company life cycle when there is a lack of historical data. It covers various scenarios and may be used as a strategic planning tool. To operate for a long time and generate profit, a company has to focus on the typical growth strategy. The natural growth strategy suits an innovator company. The "logistics" growth strategy is useful in a limited market.

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² By analogy with the epigraph to the novel by L.N. Tolstoy Anna Karenina "All happy families are alike; each unhappy family is unhappy in its own way"

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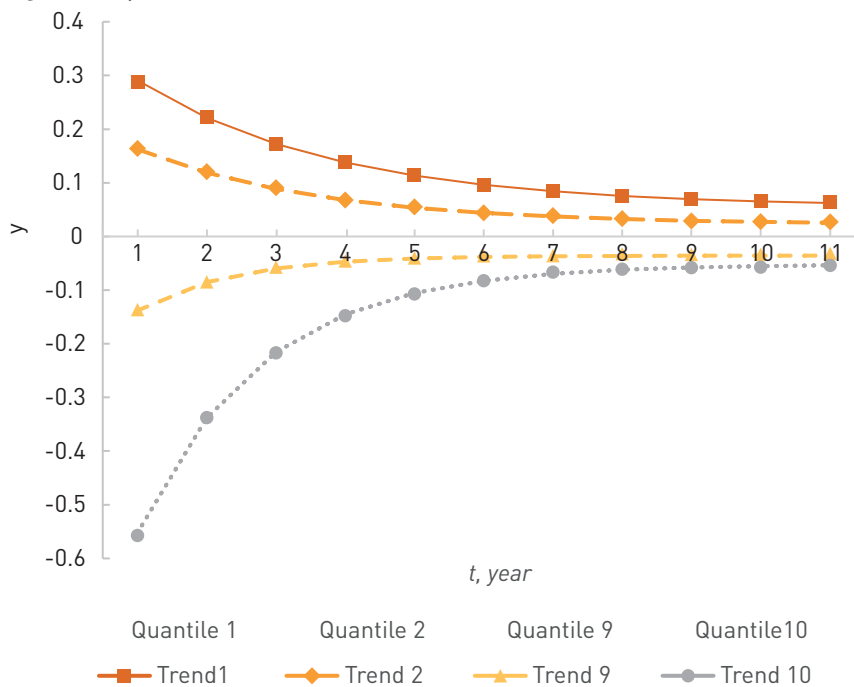
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Appendix 1

Pattern Trends

Trends of financial ratios have been studied in detail [19; 20; 22]. In the course of these studies, we compiled a sample of companies and constructed five-year or ten-year time series of financial ratios for them. Based on the zero period values, they were grouped by deciles, and group medians were subsequently analyzed. It was established that the time dynamics of the ratios' medians follow a certain pattern. One may get an idea of it by return on sales (Figure 9). Empirical data from [20] was used to construct the presented graphs, they are denoted by markers. As time passes, the medians tend to achieve the level referred to as typical [19]. According to the DuPont industry-adjusted model [20] it is representative of the industry. The greater the current deviation, the more pronounced the movement of the indicator towards the typical level. Such dynamics are indicative of the exponentiality of pattern trends, which is confirmed by the results of approximation of empirical data by functions $y = a + be^{ct}$, where y is the deviation of the median of the corresponding decile from the typical level; a, b, c – constants; t – time. In Figure 9, the results of approximation are shown as lines.

Figure 9. Dynamics of return on sales deviations



Appendix 2

Differential Trend Model

Equation (5) taking into consideration (6) is as follows:

$$\begin{aligned}
 g_t &= b_t \left\{ \left(\frac{S}{A} \right)_t \left[m_t + z_t (1 - \phi) \right] + j_t \varepsilon \right\} \left[1 + \left(\frac{D}{E} \right)_t \right] - e_t \alpha (1 - \phi) \left(\frac{D}{E} \right)_t = \\
 &= (h_0 + h_1 e^{\delta t}) \left\{ \left(f_0 + f_1 e^{\xi t} \right) \left[(c_0 + c_1 e^{\gamma t}) + (y_0 + y_1 e^{\eta t}) (1 - \phi) \right] + (n_0 + n_1 e^{\pi t}) \varepsilon \right\} \times \\
 &\times \left(1 + l_0 + l_1 e^{\lambda t} \right) - e_t \alpha (1 - \phi) (l_0 + l_1 e^{\lambda t}) = (h_0 + h_1 e^{\delta t}) \times \\
 &\times \left\{ f_0 \left[(c_0 + c_1 e^{\gamma t}) + (y_0 + y_1 e^{\eta t}) (1 - \phi) \right] + f_1 e^{\xi t} \left[(c_0 + c_1 e^{\gamma t}) + (y_0 + y_1 e^{\eta t}) (1 - \phi) \right] + \right. \\
 &\left. + n_0 \varepsilon + n_1 e^{\pi t} \varepsilon \right\} (1 + l_0 + l_1 e^{\lambda t}) - e_t \alpha (1 - \phi) (l_0 + l_1 e^{\lambda t}) = (h_0 + h_1 e^{\delta t}) \times \\
 &\times (f_0 c_0 + f_0 c_1 e^{\gamma t} + f_0 y_0 (1 - \phi) + f_0 y_1 e^{\eta t} (1 - \phi) + f_1 c_0 e^{\xi t} + f_1 c_1 e^{(\gamma + \xi)t} + f_1 y_0 (1 - \phi) e^{\xi t} +
 \end{aligned}$$

$$+ f_1 y_1 (1-\phi) e^{(\eta+\xi)t} + n_0 \varepsilon + n_1 \varepsilon e^{\pi t} \} (1+l_0 + l_1 e^{\lambda t}) - e_t \alpha (1-\phi) (l_0 + l_1 e^{\lambda t}) \}.$$

Given that $e_t = j_t = n_0 + n_1 e^{\pi t}$, we remove parentheses as follows:

$$\begin{aligned} &= (h_0 + h_1 e^{\delta t}) (f_0 c_0 (1+l_0) + f_0 c_1 (1+l_0) e^{\gamma t} + f_0 y_0 (1-\phi) (1+l_0) + f_0 y_1 (1-\phi) (1+l_0) e^{\eta t} + \\ &+ f_1 c_0 (1+l_0) e^{\xi t} + f_1 c_1 (1+l_0) e^{(\gamma+\xi)t} + f_1 y_0 (1-\phi) (1+l_0) e^{\xi t} + f_1 y_1 (1-\phi) (1+l_0) e^{(\eta+\xi)t} + \\ &+ n_0 \varepsilon (1+l_0) + n_1 \varepsilon (1+l_0) e^{\pi t} + f_0 c_0 l_1 e^{\lambda t} + f_0 c_1 l_1 e^{(\gamma+\lambda)t} + f_0 y_0 (1-\phi) l_1 e^{\lambda t} + \\ &+ f_0 y_1 (1-\phi) l_1 e^{(\eta+\lambda)t} + f_1 c_0 l_1 e^{(\xi+\lambda)t} + f_1 c_1 l_1 e^{(\gamma+\xi+\lambda)t} + f_1 y_0 (1-\phi) l_1 e^{(\xi+\lambda)t} + f_1 y_1 (1-\phi) l_1 e^{(\eta+\xi+\lambda)t} + \\ &+ n_0 \varepsilon l_1 e^{\lambda t} + n_1 \varepsilon l_1 e^{(\pi+\lambda)t} - (n_0 + n_1 e^{\pi t}) \alpha (1-\phi) l_0 - (n_0 + n_1 e^{\pi t}) \alpha (1-\phi) l_1 e^{\lambda t}) = \\ &= h_0 f_0 c_0 (1+l_0) + h_0 f_0 c_1 (1+l_0) e^{\gamma t} + h_0 f_0 y_0 (1-\phi) (1+l_0) + h_0 f_0 y_1 (1-\phi) (1+l_0) e^{\eta t} + \\ &+ h_0 f_1 c_0 (1+l_0) e^{\xi t} + h_0 f_1 c_1 (1+l_0) e^{(\gamma+\xi)t} + h_0 f_1 y_0 (1-\phi) (1+l_0) e^{\xi t} + h_0 f_1 y_1 (1-\phi) (1+l_0) e^{(\eta+\xi)t} + \\ &+ h_0 n_0 \varepsilon (1+l_0) + h_0 n_1 \varepsilon (1+l_0) e^{\pi t} + h_0 f_0 c_0 l_1 e^{\lambda t} + h_0 f_0 c_1 l_1 e^{(\gamma+\lambda)t} + h_0 f_0 y_0 (1-\phi) l_1 e^{\lambda t} + \\ &+ h_0 f_0 y_1 (1-\phi) l_1 e^{(\eta+\lambda)t} + h_0 f_1 c_0 l_1 e^{(\xi+\lambda)t} + h_0 f_1 c_1 l_1 e^{(\gamma+\xi+\lambda)t} + h_0 f_1 y_0 (1-\phi) l_1 e^{(\xi+\lambda)t} + \\ &+ h_0 f_1 y_1 (1-\phi) l_1 e^{(\eta+\xi+\lambda)t} + h_0 n_0 \varepsilon l_1 e^{\lambda t} + h_0 n_1 \varepsilon l_1 e^{(\pi+\lambda)t} - \\ &- n_0 \alpha (1-\phi) h_0 l_0 - n_0 \alpha (1-\phi) h_0 l_1 e^{\lambda t} - n_1 \alpha (1-\phi) h_0 l_0 e^{\pi t} - n_1 \alpha (1-\phi) h_0 l_1 e^{(\pi+\lambda)t} + \\ &+ h_1 f_0 c_0 (1+l_0) e^{\delta t} + h_1 f_0 c_1 (1+l_0) e^{(\gamma+\delta)t} + h_1 f_0 y_0 (1-\phi) (1+l_0) e^{\delta t} + h_1 f_0 y_1 (1-\phi) (1+l_0) e^{(\eta+\delta)t} + \\ &+ h_1 f_1 y_1 (1-\phi) (1+l_0) e^{(\eta+\xi+\delta)t} + h_1 n_0 \varepsilon (1+l_0) e^{\delta t} + h_1 n_1 \varepsilon (1+l_0) e^{(\pi+\delta)t} + h_1 f_0 c_0 l_1 e^{(\lambda+\delta)t} + \\ &+ h_1 f_0 c_1 l_1 e^{(\gamma+\lambda+\delta)t} + h_1 f_0 y_0 (1-\phi) l_1 e^{(\lambda+\delta)t} + h_1 f_0 y_1 (1-\phi) l_1 e^{(\eta+\lambda+\delta)t} + h_1 f_1 c_0 l_1 e^{(\xi+\lambda+\delta)t} + \\ &+ h_1 f_1 c_1 l_1 e^{(\gamma+\xi+\lambda+\delta)t} + h_1 f_1 y_0 (1-\phi) l_1 e^{(\xi+\lambda+\delta)t} + h_1 f_1 y_1 (1-\phi) l_1 e^{(\eta+\xi+\lambda+\delta)t} + h_1 n_0 \varepsilon l_1 e^{(\lambda+\delta)t} + \\ &+ h_1 n_1 \varepsilon l_1 e^{(\pi+\lambda+\delta)t} - n_0 \alpha (1-\phi) h_1 l_0 e^{\delta t} - n_0 \alpha (1-\phi) h_1 l_1 e^{(\lambda+\delta)t} - \\ &- n_1 \alpha (1-\phi) h_1 l_0 e^{(\pi+\delta)t} - n_1 \alpha (1-\phi) h_1 l_1 e^{(\pi+\lambda+\delta)t}. \end{aligned}$$

Hence, we group summands with the exponent equal powers and obtain the following:

$$\begin{aligned} g_t &= h_0 f_0 c_0 (1+l_0) + h_0 f_0 y_0 (1-\phi) (1+l_0) + h_0 n_0 \varepsilon (1+l_0) - n_0 \alpha (1-\phi) h_0 l_0 + \\ &+ h_1 f_0 c_0 (1+l_0) e^{\delta t} + h_1 f_0 y_0 (1-\phi) (1+l_0) e^{\delta t} + h_1 n_0 \varepsilon (1+l_0) e^{\delta t} - n_0 \alpha (1-\phi) h_1 l_0 e^{\delta t} + \\ &+ h_0 f_0 c_1 (1+l_0) e^{\gamma t} + \\ &+ h_0 f_0 y_1 (1-\phi) (1+l_0) e^{\eta t} + \\ &+ h_0 n_1 \varepsilon (1+l_0) e^{\pi t} - n_1 \alpha (1-\phi) h_0 l_0 e^{\pi t} + \\ &+ h_0 f_1 c_0 (1+l_0) e^{\xi t} + h_0 f_1 y_0 (1-\phi) (1+l_0) e^{\xi t} + \\ &+ h_0 f_0 c_0 l_1 e^{\lambda t} + h_0 f_0 y_0 (1-\phi) l_1 e^{\lambda t} + h_0 n_0 \varepsilon l_1 e^{\lambda t} - n_0 \alpha (1-\phi) h_0 l_1 e^{\lambda t} + \\ &+ h_1 f_0 c_1 (1+l_0) e^{(\gamma+\delta)t} + \\ &+ h_0 f_0 c_1 l_1 e^{(\gamma+\lambda)t} + \\ &+ h_1 f_0 c_0 l_1 e^{(\lambda+\delta)t} + h_1 f_0 y_0 (1-\phi) l_1 e^{(\lambda+\delta)t} + h_1 n_0 \varepsilon l_1 e^{(\lambda+\delta)t} - n_0 \alpha (1-\phi) h_1 l_1 e^{(\lambda+\delta)t} + \\ &+ h_0 f_0 y_1 (1-\phi) l_1 e^{(\eta+\lambda)t} + \end{aligned}$$

$$\begin{aligned}
& +h_0n_1\varepsilon l_1e^{(\pi+\lambda)t} - n_1\alpha(1-\phi)h_0l_1e^{(\pi+\lambda)t} + \\
& +h_0f_1c_1(1+l_0)e^{(\gamma+\xi)t} + \\
& +h_1f_1c_0(1+l_0)e^{(\xi+\delta)t} + h_1f_1y_0(1-\phi)(1+l_0)e^{(\xi+\delta)t} + \\
& +h_0f_1y_1(1-\phi)(1+l_0)e^{(\eta+\xi)t} + \\
& +h_0f_1c_0l_1e^{(\xi+\lambda)t} + h_0f_1y_0(1-\phi)l_1e^{(\xi+\lambda)t} + \\
& +h_1f_0y_1(1-\phi)(1+l_0)e^{(\eta+\delta)t} + \\
& +h_1n_1\varepsilon(1+l_0)e^{(\pi+\delta)t} - n_1\alpha(1-\phi)h_1l_0e^{(\pi+\delta)t} + \\
& +h_1f_0c_1l_1e^{(\gamma+\lambda+\delta)t} + \\
& +h_1f_1c_1(1+l_0)e^{(\gamma+\xi+\delta)t} + \\
& +h_1f_1c_0l_1e^{(\xi+\lambda+\delta)t} + h_1f_1y_0(1-\phi)l_1e^{(\xi+\lambda+\delta)t} + \\
& +h_0f_1y_1(1-\phi)l_1e^{(\eta+\xi+\lambda)t} + \\
& +h_0f_1c_1l_1e^{(\gamma+\xi+\lambda)t} + \\
& +h_1n_1\varepsilon l_1e^{(\pi+\lambda+\delta)t} - n_1\alpha(1-\phi)h_1l_1e^{(\pi+\lambda+\delta)t} + \\
& +h_1f_1y_1(1-\phi)(1+l_0)e^{(\eta+\xi+\delta)t} + \\
& +h_1f_0y_1(1-\phi)l_1e^{(\eta+\lambda+\delta)t} + \\
& +h_1f_1c_1l_1e^{(\gamma+\xi+\lambda+\delta)t} + \\
& +h_1f_1y_1(1-\phi)l_1e^{(\eta+\xi+\lambda+\delta)t} .
\end{aligned}$$

Appendix 3

Initial Data for Modeling

The increment rates and sustainable growth trajectories (see Figures 1 and 2), the "Logistic" sustainable growth trajectory (see Figure 7) and the growth trajectory with a decline period (see Figure 8) were modeled using the financial ratios indicated in Tables 1–3.

Table 1. Initial data for modeling the increment rates and sustainable growth trajectories

Ratio	Growth			
	with spread	without spread	catching-up	industry
b	0.7	0.7	0.2	0.7
S/A	1.5	1.5	1.5	1.5
m	20%	20%	10%	6%
z	10%	0%	-5%	0%
φ	22%	22%	22%	22%
j	9%	9%	9%	9%
ε	0.7	0.7	0.7	0.7
D/E	1.0	1.0	1.0	1.0

Table 2. Initial data for modeling the "logistic" trajectory of sustainable growth

Ratio	$t = 0$	$t = \infty$
b	0.7	0.7
S/A	1.5	1.5
m	68%	0%
z	10%	0%
ϕ	22%	22%
j	9%	0%
ε	0.7	0.7
D/E	1.0	0.0

Table 3. Initial data for modeling the growth trajectory with a decline period

Ratio	$t = 0$	$t = \infty$
b	0.7	0.7
S/A	1.5	1.5
m	48%	-5%
z	10%	0%
ϕ	22%	22%
j	9%	9%
ε	0.7	0.7
D/E	1.0	1.0

The values of financial ratios correspond to the average levels, which emerged in 2001–2021. A company engaged in extraction of commercial minerals was the prototype. The share of debt with a free-floating interest rate α is taken to be equal to 100%. The cost of debt and inflation of manufacturing resource price correspond to general economic inflation.