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Internal Inconsistency of Downside CAPM models

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There is little criticism of downside measures approach to estimating risk premiums. Instead, this model attracted attention of both academics and practitioners. For example, Abbas et al (2011) refer to DCAPM as "long-awaited solution for asset pricing problem". Estrada (2006, 2007), Post and Vliet (2004) and some other researchers claim that downside approach is often preferable to the traditional CAPM. At the same time they keep silence about limitations of the model. This paper intends to fill the gap. It shows that downside CAPM approach produces instable results. Although there are various versions of downside risk measurement approaches, the focus of this paper is estimation of assets' betas and risk premiums. Therefore we limit our review with DCAPM approach, which is the most popular today and is advised as a preferable method of estimating risk premiums at emerging markets. In this context the most critical issue is estimation of security characteristic line and the marginal contribution of an asset's risk into the risk of benchmarking market portfolio.

We construct a portfolio of two assets using different weights and calculate its true semideviation and semideviation according to DCAPM framework. Then we change data to obtain scenarios with different correlations between asset returns and repeat simulation across different weights with new data. The simulation shows that if the assets are perfectly positively correlated the DCAPM's estimation percentage error is zero, and if the assets are perfectly negatively correlated the DCAPM's estimation percentage error can approach infinity. If the assets have zero correlation, the percentage error is quite significant (may reach 15% in the simulation under appropriate weights). The magnitude of the estimation error varies both with correlation between data, weights of the assets and the data themselves. So the algorithm cannot provide reliable estimates. Such results imply that downside beta is unreliable measure of security systematic risk.

1. Introduction

One of the most important issues in practical finance is estimation of risk premium of an asset. Business valuation and project appraisal to a considerable degree hinge on the appropriately estimated cost of capital and can be misleading if the estimation of risk premium was inaccurate. Therefore the quest for consistent, practical and accurate models for finding risk premium of an asset from observable market data is still on agenda. In spite of continuing debate of academics, the most popular model in practice is CAPM. Perhaps, the major reason of CAPM's popularity is that it is easy to understand and simple to apply. However CAPM relies on a set of unrealistic assumptions and is valid only for the ideal case of symmetric normally distributed variables with constant variation. Many researchers (Hwang and Satchell, 1999; Harvey, 2011; Claessens, Dasgupta, Glen, 1998; Serra, 2003; Teplova and Shutova, 2010) cast doubt on suitability of classical CAPM approach for emerging markets, where returns are highly non-normal and the variation is time-dependent. Therefore emerging markets do not suite classical mean-variance world assumptions. Attempts to improve the CAPM model are ongoing. A lot of complicated theoretical modification has appeared in recent decades. However most of the complicated academic models are not easy to apply in practice. Vaihekoski (2005) provides a review of major applied modifications of CAPM and tests the most influential alternative approaches. He discovered the considerable gap between

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ex-ante approach to estimating market risk premium and CAPM-based approaches.

Recently, approaches based on downside risk measures enjoyed wide popularity. It is considered that downside risk measures are more relevant for assessment of risk and asset pricing than traditional mean-variance approach, as investors primarily concern about possibility of losses and therefore should consider lower partial moments (LPMs) of probability distribution of an asset's returns. This approach is sometimes called mean-LPM portfolio theory (Kong, 2006; Briec, and Kerstens, 2007; Wojt, 2009). Nawrocki (1999) and Ang et al (2005) provide a concise history of downside risk measures. For a further discussion of downside risk measures see Breitmeyer, Hakenes and Pfingsten (2001), Nawrocki (2003).

Although this new approach relaxes the restrictive assumption about normality of asset returns, which is essential in classic mean-variance framework, some researchers challenge its consistency with utility theory. For example, Alexander (2008, p. 267) notes that popular downside risk adjusted performance measures such as kappa index, Sortino ratio and omega statistic "are either not linked to a utility function at all, or if they are associated with a utility function we assume the investor cares nothing at all about the gains he makes above a certain threshold". She states that any utility associated with LPM risk measures "will be a strange type of utility, because it can be anything at all for 'upside' returns that are above the threshold" (Alexander, 2008, p. 265). There are no sufficient theoretical justifications for such a type of utility functions. Therefore LPM measures should be regarded as approximate estimates of risk for situations of non-normally distributed returns, where traditional risk measures turn out to be non-robust and unreliable.

Since the problem of heavy tails in the probability distributions of assets' returns strongly influences the applicability of classical CAPM framework, researchers suggested substituting volatility of returns with LPM risk measures. Hogan and Warren (1974), Bawa and Lindenberg (1977), Harlow and Rao (1989) suggested variants of CAPM based on LPM measures of risk.

Recently, Estrada (2000, 2001, 2002, 2003) suggested an "approximately correct" mean-semivariance behavior criterion for skewed probability distributions of returns and constructed a Downside CAPM model. Estrada's approach is based on semideviation, which is the simplest LPM measure. Harry Markowitz (1959) in his groundbreaking "Portfolio Selection" stated that the semideviation produces efficient portfolios somewhat preferable to those of the standard deviation. Estrada (2003) noted that the reason for neglecting by Markowitz the downside measure of risk in subsequent analysis were that the semideviation was a relatively unknown measure of risk and mean-semivariance portfolios were difficult to obtain then. In times of Markowitz the CAPM wasn't known yet. The purpose of Estrada's DCAPM framework is to find an appropriate beta and risk premium for a given asset. Although this model is not indisputable, it became quite popular. Relying on results of statistical research of stock returns in 28 emerging markets Estrada (2002) concluded that his DCAPM framework is more suitable for emerging markets. There is growing empirical research of emerging markets based on DCAPM framework (Galagedera and Brooks, 2005; Ang, Chen and Xing, 2005; Teplova, 2005, 2007a, 2007b, 2011; Bukhvalov and Okulov, 2006; Beach, 2006; Galagedera, 2007; Soares da Silva, 2007; Lee, Reed and Robinson, 2008; Brandouy, Kerstens and Woestyne, 2009; Tsonchev and Kostenarov, 2009; Cwynar,and Kazmierkiewicz, 2010; Artavanis, Diacogiannis and Mylonakis, 2010; Abbas et al, 2011). Researchers report controversial results of statistical tests. However the Estrada's model has become well-known. Recently Amirhosseini, Roodposhti and Ghobadi (2011) introduced conditional DCAPM model, tested it for 70 Tehran stock exchange companies during 2002-2008 and reported it has higher explaining ability than classic CAPM.

Surprisingly, there is little criticism of downside measures approach to estimating risk premiums (Varga-Haszonits and Imre Kondor, 2008). Instead, this model attracted attention of both academics and practitioners. For example, Abbas et al (2011) refer to DCAPM as "long-awaited solution for asset pricing problem". Estrada (2006, 2007), Post and Vliet (2004) and some other researchers claim that downside approach is often preferable to the traditional CAPM. At the same time they keep silence about limitations of the model. This paper intends to fill the gap. It shows that downside CAPM approach produces instable results. Although there are various versions of downside risk measurement approaches, the focus of this paper is estimation of assets' betas and risk premiums. Therefore we limit our review with DCAPM approach, which is the most popular today and is advised as a preferable method of estimating risk premiums at emerging markets. In this context the most critical issue is estimation of security characteristic line and the marginal contribution of an asset's risk into the risk of benchmarking market portfolio. This implies the need to take into account co-skewness of assets' returns (Harvey and Siddique, 2000). Downside CAPM approach intends to measure this co-skewness by means of co-semivariance statistic. However to be valid downside CAPM should assume constant and exogenous co-semivariation. Although semivariance measures are useful and correct, the formula for calculating the co-semivariance (downside correlation) is a questionable statistic that cannot represent real dependencies between the two assets. This measure ignores the ability of upside returns of one asset to hedge the downside returns of another asset in a portfolio. Therefore, the CAPM based on instable and controversial co-semivariance can be highly misleading. An appropriate measure of covariance is an essential component of CAPM. As a result the DCAPM model does not appropriately account for true systematic risk of an asset and cannot produce reliable estimates of an asset's beta and risk premium.

In contrast with other researchers who rely on statistical tests of empirical data, we address to the simulation of stylized data, for which it is possible to calculate true semideviation of a portfolio. We construct a portfolio of two assets using different weights and calculate its true semideviation and semideviation according to DCAPM framework. Then we change data to obtain scenarios with different correlations between asset returns and repeat simulation across different weights with new data. The simulation shows that if the assets are perfectly positively correlated the DCAPM's estimation percentage error is zero, and if the assets are perfectly negatively correlated the DCAPM's estimation percentage error can approach infinity. If the assets have zero correlation, the percentage error is quite significant (may reach 15% in the simulation under appropriate weights). The magnitude of the estimation error varies both with correlation between data, weights of the assets and the data themselves. Such results imply that downside beta is unreliable measure of security systematic risk. CAPM based on such instable and controversial measure generally can be misleading.

Within classic CAPM standard deviation of an asset returns, standard deviation of the market returns, correlation between the asset and the market are all constant statistics at a given moment of time and do not depend on the weight of the asset in the market portfolio. Instead within DCAPM framework the measure of semi-correlation is questionable, because dependence between downside disturbances of returns of two assets is not constant and depends on the weight of the asset in the market portfolio and traditional correlation between them. Semi-correlation completely ignores the possibility of upside returns to compensate downside returns, therefore semi-correlation distorts true relationship between returns of two variables. Hence downside beta does not accurately estimate the contribution of an asset to the risk of the market portfolio. Even though DCAPM is positioned as "approximately correct" approach it intends to improve traditional CAPM, which is based on such "a questionable and restrictive measure of risk" (Estrada, 2007b, p. 170) as variance, but it relies on even more questionable measure of co-semivariance. We need further research for more appropriate measures of systematic risk.

2. Internal Inconsistencies of DCAPM Model

The major component of downside CAPM model is downside beta. Therefore the validity of the downside beta rests on the correctness of the cosemivariance formula. But as we will see below the cosemivariance formula is evidently questionable and unreliable in portfolio selection.

Estrada (2003) suggested that the downside covariance (co-semivariance for short) is given by

$$CSVar_{AB} = \frac{1}{T}\sum\left\{Min\left(R_A - \mu_A, 0\right)\cdot Min\left(R_B - \mu_B, 0\right)\right\}, \tag{1}$$

where $CSVar_{A\,B}$ – co-semivariance between asset A and asset B;
R_A – return on asset A at a moment of time;
R_B – return on asset B at a moment of time;
μ_A – mean return of asset A;
μ_B – mean return of asset B;
T – number of observations.
Then to obtain scale-independence he proposes the downside correlation formula:

$$Scorrel = \frac{CSVar_{A\,B}}{\sigma_A^D \cdot \sigma_B^D} = \frac{\frac{1}{T}\sum\left\{Min\left(R_A-\mu_A,0\right)\cdot Min\left(R_B-\mu_B,0\right)\right\}}{\sqrt{\frac{1}{T}\sum\left\{Min\left(R_A-\mu_A,0\right)^2\right\}\cdot\frac{1}{T}\sum\left\{Min\left(R_B-\mu_B,0\right)^2\right\}}}, \quad (2)$$

where σ_A^D – downside standard deviation of returns of asset A;
σ_B^D – downside standard deviation of returns of asset B.

The problem is that the concepts of cosemivariance and downside correlation introduced in formulas (1) and (2) by definition imply the possibility to estimate unambiguous relationship between downside disturbances of two variables. But there is no stable relationship between downside disturbances, because downside disturbances of one variable can be partly or completely canceled by upside disturbances of another variable. The degree to which upside disturbances of the second variable cancel downside disturbances of the first variable is not constant, but depends on the proportions of variables taken together, correlation between variables and other characteristics of joint distribution. By ignoring the possibility of upside returns to compensate downside returns the proposed measure, which is called downside correlation, cannot estimate correct dependences between downside disturbances of two variables. Both construction of this measure and its outcomes are doubtful. Especially for the portfolio diversification purposes the upside and downside dependences are also important and should be considered as well as the downside dependences. There is no sufficient research and theoretical justifications for cosemivariance and semicorrelation. This statistic is not well-founded and it is questionable whether it accurately corresponds to the real world. Particularly, even for the case of two normally distributed variables there are no verifiable values for downside relationship, which allow estimate the accuracy of downside correlation, because in fact there is no linear dependence between downside disturbances or between upside disturbances. Moreover, the measure of downside correlation is not relevant even for extreme controlling cases, when there is perfect negative correlation between two variables, perfect positive correlation and zero correlation. Examples below will illustrate these points in more detail.

There is no need in strict mathematical proofs for casting doubt on the formulas (1) and (2). The simple illustrative example will be sufficient. It is known that statistic inference relies on inductive reasoning. And logical induction cannot guarantee truth of a conclusion. It allows only formulate hypothesis, and inductive arguments are always prone to weaknesses. The argument that if all swans we have seen are white, and therefore all swans are white, is obviously falsifiable, because logically it is possible that there can be found a swan which is not white (Popper, 2002). In contrast, only one disconfirming instance is sufficient to disprove a theory or a statistical hypothesis, if this instance contradicts predictions of that hypothesis. Therefore, it would be useful to examine the performance of DCAPM's algorithm by testing it on simple verifiable examples with known assumptions.

The table 1 presents the true algorithm for calculating the portfolio's semideviation. For illustrative purposes and to avoid unnecessary complications a 10 year example was taken from Estrada (2006). The tables are self-explanatory. The portfolio was constructed using weight 0.3 for

Asset A and 0.7 for asset B (the market).

Table 1

Portfolio Semideviation Calculation – Basic Scenario

Weights					
A	0.3				
B	0.7				
Year	R_A	R_B	Portfolio	min(R_p-μ,0)	min(R_p-μ,0)^2
1995	44.0%	37.6%	39.5%	0.0%	0,0000
1996	47.8%	23.0%	30.4%	0.0%	0,0000
1997	-19.8%	33.4%	17.4%	-4.7%	0,0022
1998	93.3%	28.6%	48.0%	0.0%	0,0000
1999	289.8%	21.0%	101.6%	0.0%	0,0000
2000	3.7%	-9.1%	-5.3%	-27.4%	0,0750
2001	-52.5%	-11.9%	-24.1%	-46.2%	0,2135
2002	-21.8%	-22.1%	-22.0%	-44.1%	0,1948
2003	22.5%	28.7%	26.8%	0.0%	0,0000
2004	3.7%	10.9%	8.7%	-13.4%	0,0179
Average			**22.1%**	**-13.6%**	**5,0%**
Semideviation					**22.4%**

In Table 2 Estrada's formulas were applied in calculations of downside semivariance and downside correlation. Figure 1 presents downside security characteristic line. This figure is also adapted from Estrada (2006). The estimated downside beta is 2.25. Below we will estimate the portfolio semivariance using co-semivariance statistic from table 2 and compare the result with the true portfolio semivariance calculated in table 1.

Table 2

Estrada's DCAPM framework measures calculation – Basic Scenario

Year	Ra	Rb	min(R_A-μ,0)	min(R_B-μ,0)	min(R_A-μ,0)^2	min(R_B-μ,0)^2	min(R_A-μ,0)*min(R_B-μ,0)
1995	44.0%	37.6%	0.0%	0.0%	0.0000	0.0000	0,0%
1996	47.8%	23.0%	0.0%	0.0%	0.0000	0.0000	0,0%
1997	-19.8%	33.4%	-60.9%	0.0%	0.3705	0.0000	0,0%
1998	93.3%	28.6%	0.0%	0.0%	0.0000	0.0000	0,0%
1999	289.8%	21.0%	0.0%	0.0%	0.0000	0.0000	0,0%
2000	3.7%	-9.1%	-37.4%	-23.1%	0.1397	0.0534	8,6%
2001	-52.5%	-11.9%	-93.6%	-25.9%	0.8755	0.0671	24,2%
2002	-21.8%	-22.1%	-62.9%	-36.1%	0.3953	0.1304	22,7%
2003	22.5%	28.7%	-18.6%	0.0%	0.0345	0.0000	0,0%
2004	3.7%	10.9%	-37.4%	-3.1%	0.1397	0.0010	1,2%
Average	**41.1%**	**14.0%**			**19.6%**	**2.5%**	
Semivariance					**44.2%**	**15.9%**	
Co-semivariance							**0.0567**
Downside Correlation							**0.81**
Downside Beta							**2.25**

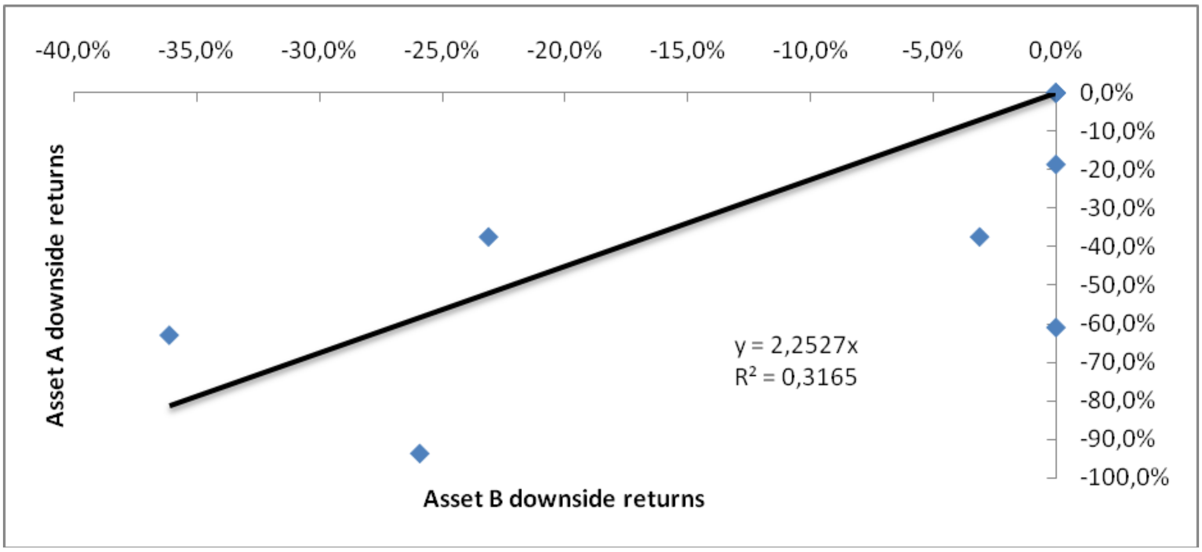

Figure 1. Downside Security Characteristic Line

Within DCAPM framework Estrada proposes the traditional way to find the portfolio's downside semideviation using the following formula:

$$\sigma_p^D = \sqrt{w_A{}^2\sigma_A^{D2} + w_B{}^2\sigma_B^{D2} + 2w_A w_B \rho^D \sigma_A^D \sigma_B^D}, \tag{3}$$

where σ_p^D – portfolio downside semideviation;

σ_A^D – asset A downside semideviation;

σ_B^D – asset B downside semideviation;

ρ^D – downside correlation between assets A and B;

w_A – weight of asset A in the portfolio;

w_B – weight of asset B in the portfolio.

The formula of portfolio downside semivariance was adopted from classic mean-variance framework, which relies on restrictive assumptions of symmetrical normally distributed variables. Under assumptions of mean-variance framework portfolio return variance is a linear combination of the variances and co-variances of returns of each asset in that portfolio. Variance (and correspondently standard deviation) is not a robust statistic; instead variance is strongly affected by outliers and small departures from model assumptions of normality can produce significant distortions when distribution is not normal. The introduction of portfolio return semivariance was intended to improve this weakness. However this property of portfolio variance does not hold for portfolio semivariance. Finding portfolio return semivariance requires quite complicated algorithms (Kong, 2006; Cumova and Nawrocki, 2011). There is no simple analytical solution to compute the LPM or UPM of a portfolio (Wojt, 2009). Hence, formula (3) is not justified for calculations of portfolio downside semivariance. It can be regarded only as a simplified approximation of real dependences between downside returns of two assets.

To illustrate the performance of (3), let us put the numbers from Table 2 into formula (3), using weight 0.3 for Asset A and 0.7 for asset B:

$$\sigma_p^D = \sqrt{0.3^2 \cdot 0.442^2 + 0.7^2 \cdot 0.159^2 + 2 \cdot 0.3 \cdot 0.7 \cdot 0.81 \cdot 0.442 \cdot 0.159} = 23.2\%.$$

The result is slightly different of the true 22.4% of the portfolio's semideviation. The percentage error of this estimation is around 3.57%. Percentage error is calculated as estimate minus the true value divided by the true value and multiplied by 100. In the above case it seems to be insignificant; however the accuracy of approximation may vary with data and weights of assets in a portfolio. Estrada (2007) himself recognizes the problem. He wrotes: "the semicovariance matrix is endogenous; that is, a change in weights affects the periods in which the portfolio underperforms the benchmark, which in turn affects the elements of the semicovariance matrix… If instead of the semideviation of one portfolio we wanted to calculate the portfolio with the lowest semideviation from a set of, say, 1,000 feasible portfolios, we would first need to calculate the returns of each portfolio; then from those returns we would need to calculate the semideviation of each portfolio; and finally from those semideviations we would need to select the one with the lowest value" (Estrada, 2007a). In order to overcome this problem Estrada offered a heuristic approach, shortly described above in (1), "which generates a symmetric and exogenous semicovariance matrix" (Estrada, 2007a). But this approach is only approximation, not an exact analytical solution.

Actually, it may be practical to solve the problem with a sufficiently accurate heuristic algorithm. However the crux of the matter is in the accuracy of approximation. Even the simple sensitivity analysis demonstrates that the accuracy of DCAPM's estimates for a 2 assets portfolio's semideviation is questionable. The percentage error of the estimation can vary from 0 to ∞, depending on the data inputs and weights of assets in a portfolio. At the extreme, when the downside returns in one asset are fully compensated by upside returns of another asset, the DCAPM's estimation percentage error tends to infinity. For example, for two perfectly negatively correlated assets we can construct such a portfolio, in which upside returns of one asset will completely cancel downside returns of another asset. As a result both variance and downside variance of that portfolio return will be zero. Since measure of downside correlation introduces in (2) completely ignores the possibility of upside returns of one asset to compensate downside returns of another asset, DCAPM algorithm produces significant inaccuracy for the cases of assets with negative and low correlation.

Let us see several scenarios of data inputs and corresponding results of sensitivity analysis based on the model, provided in Estrada (2006). These scenarios may be interpreted as disconfirming instances, as they demonstrate the possibility of significant estimation errors.

For example, in the scenario 2, presented in Tables 3 and 4, the difference between value of portfolio's semideviation estimated by means of DCAPM framework and the exact value is 1.6% (percentage error 9.7%).

Table 3

Portfolio Semideviation Calculation – Second Scenario

Weights					
A	0.3				
B	0.7				
Year	Ra	Rb	Portfolio	min(rp-mu,0)	min(rp-mu,0)^2
1995	44.0%	37.6%	39.5%	0.0%	0.0000
1996	47.8%	23.0%	30.4%	0.0%	0.0000
1997	-19.8%	33.4%	17.4%	-10.2%	0.0105
1998	93.3%	28.6%	48.0%	0.0%	0.0000
1999	289.8%	21.0%	101.6%	0.0%	0.0000
2000	13.7%	-9.1%	-2.3%	-29.9%	0.0897
2001	12.5%	-11.9%	-4.6%	-32.3%	0.1041
2002	-21.8%	25.1%	11.0%	-16.7%	0.0277
2003	22.5%	28.7%	26.8%	-0.8%	0.0001
2004	3.7%	10.9%	8.7%	-18.9%	0.0359
Average			**27.7%**	**-10.9%**	**2.7%**
Semideviation					**16.4%**

Table 4

Estrada DCAPM framework measures calculation – Second Scenario

Year	Ra	Rb	min(ra-mu,0)	min(rb-mu,0)	min(ra-mu,0)^2	min(rb-mu,0)^2	min(ra-mu,0)*min(rb-mu,0)
1995	44.0%	37.6%	-4.6%	0.0%	0.0021	0.0000	0,0%
1996	47.8%	23.0%	-0.8%	0.0%	0.0001	0.0000	0,0%
1997	-19.8%	33.4%	-68.4%	0.0%	0.4674	0.0000	0,0%
1998	93.3%	28.6%	0.0%	0.0%	0.0000	0.0000	0,0%
1999	289.8%	21.0%	0.0%	0.0%	0.0000	0.0000	0,0%
2000	13.7%	-9.1%	-34.9%	-27.8%	0.1216	0.0775	9,7%
2001	12.5%	-11.9%	-36.1%	-30.6%	0.1301	0.0938	11,0%
2002	-21.8%	25.1%	-70.4%	0.0%	0.4952	0.0000	0,0%
2003	22.5%	28.7%	-26.1%	0.0%	0.0680	0.0000	0,0%
2004	3.7%	10.9%	-44.9%	-7.8%	0.2013	0.0061	3,5%
Average	**48.6%**	**18.7%**			**14.9%**	**1.8%**	
Semivariance					**38.5%**	**13.3%**	
Cosemivariance							**0.0243**
Downside Correlation							**0.47**
Downside Beta							**1.37**

Figures 2 and 3 present the results of a sensitivity analysis for data used in scenario 2. To obtain graphs we changed weight of asset A from 0 to 1. Weight of asset B is calculates as 1 minus weight of asset A.

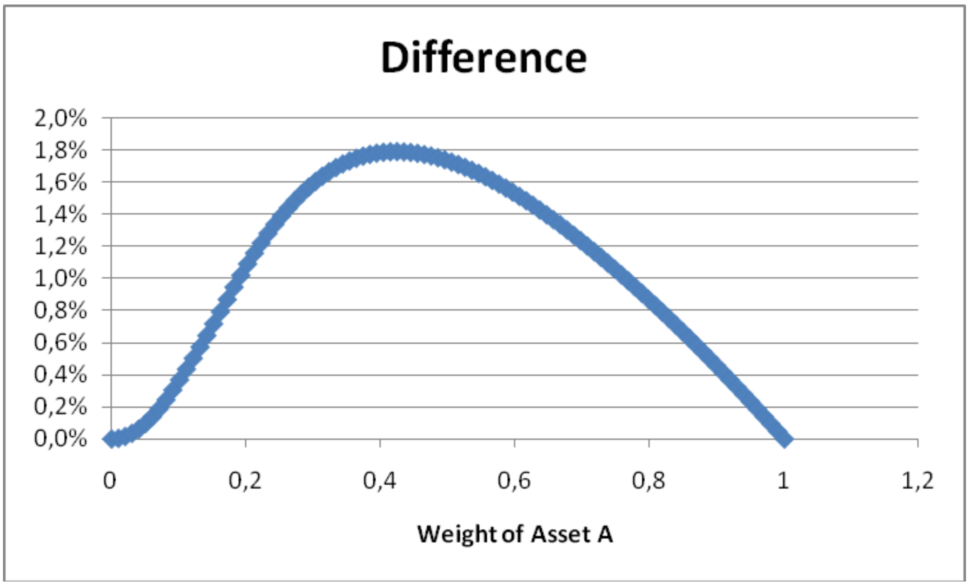

Figure 2. Sensitivity analysis of absolute estimation error under different weights of assets in a portfolio

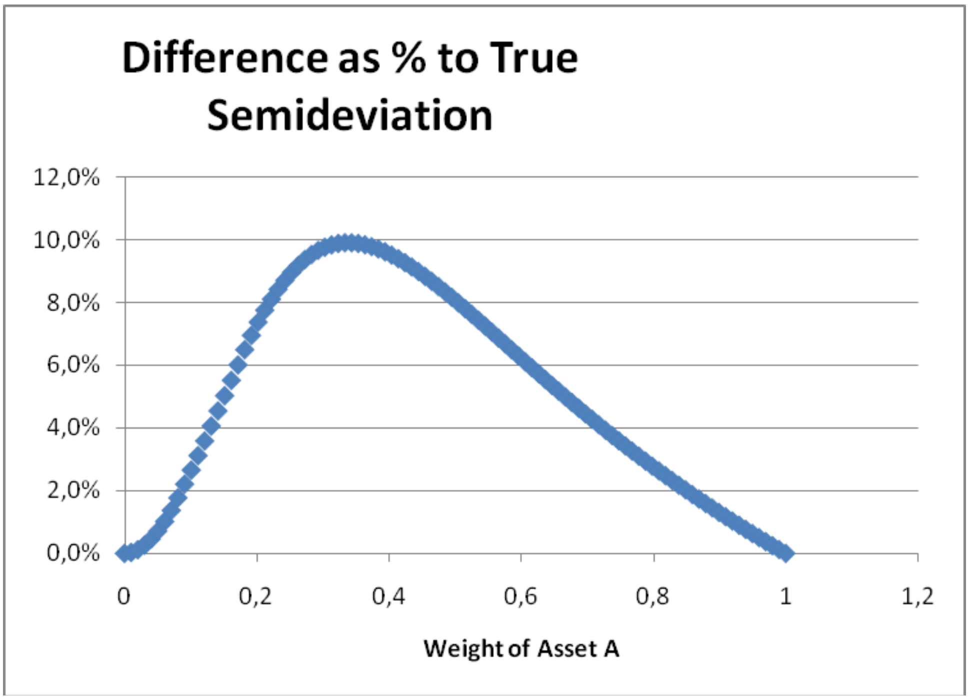

Figure 3. Sensitivity analysis of percentage estimation error under different weights of assets in a portfolio

The scenario 3 is obtained after slightly changing the data. According to Tables 5 and 6 the difference between value of the portfolio's semideviation estimated by means of DCAPM framework and the exact value is 10.6% or 71.9% as a percent to the exact value.

Table 5

Portfolio Semideviation Calculation – Third Scenario

Weights					
A	0.3				
B	0.7				
Year	Ra	Rb	Portfolio	min(rp-mu,0)	min(rp-mu,0)^2
1995	44.0%	37.6%	39.5%	-10.3%	0,0105
1996	55.0%	23.0%	32.6%	-17.2%	0,0295
1997	-80.0%	140.0%	74.0%	0.0%	0,0000
1998	93.3%	28.6%	48.0%	-1.8%	0,0003
1999	180.0%	21.0%	68.7%	0.0%	0,0000
2000	13.7%	78.0%	58.7%	0.0%	0,0000
2001	78.0%	-5.0%	19.9%	-29.9%	0,0892
2002	-29.0%	125.0%	78.8%	0.0%	0,0000
2003	17.0%	75.0%	57.6%	0.0%	0,0000
2004	78.0%	-5.0%	19.9%	-29.9%	0,0892
Average			**49.8%**	**-8.9%**	**2,2%**
Semideviation					**14.8%**

Table 6

Estrada DCAPM framework measures calculation – Third Scenario

Year	Ra	Rb	min(ra-mu,0)	min(rb-mu,0)	min(ra-mu,0)^2	min(rb-mu,0)^2	min(ra-mu,0)*min(rb-mu,0)
1995	44.0%	37.6%	-1.0%	-14.2%	0.0001	0.0202	0,1%
1996	55.0%	23.0%	0.0%	-28.8%	0.0000	0.0831	0,0%
1997	-80.0%	140.0%	-125.0%	0.0%	1.5625	0.0000	0,0%
1998	93.3%	28.6%	0.0%	-23.2%	0.0000	0.0539	0,0%
1999	180.0%	21.0%	0.0%	-30.8%	0.0000	0.0950	0,0%
2000	13.7%	78.0%	-31.3%	0.0%	0.0980	0.0000	0,0%
2001	78.0%	-5.0%	0.0%	-56.8%	0.0000	0.3229	0,0%
2002	-29.0%	125.0%	-74.0%	0.0%	0.5476	0.0000	0,0%
2003	17.0%	75.0%	-28.0%	0.0%	0.0784	0.0000	0,0%
2004	78.0%	-5.0%	0.0%	-56.8%	0.0000	0.3229	0,0%
Average	**45.0%**	**51.8%**			**22.9%**	**9.0%**	
Semideviation					**47.8%**	**30.0%**	
Cosemivariance							**0.0001**
Downside Correlation							**0.00**
Downside Beta							**0.00**

Figures 4-6 demonstrate the results of sensitivity analysis under data choosen in scenario 3. According to figure 6 the percentage error of estimation may approach to 222%, when weight of A is 0.484848.

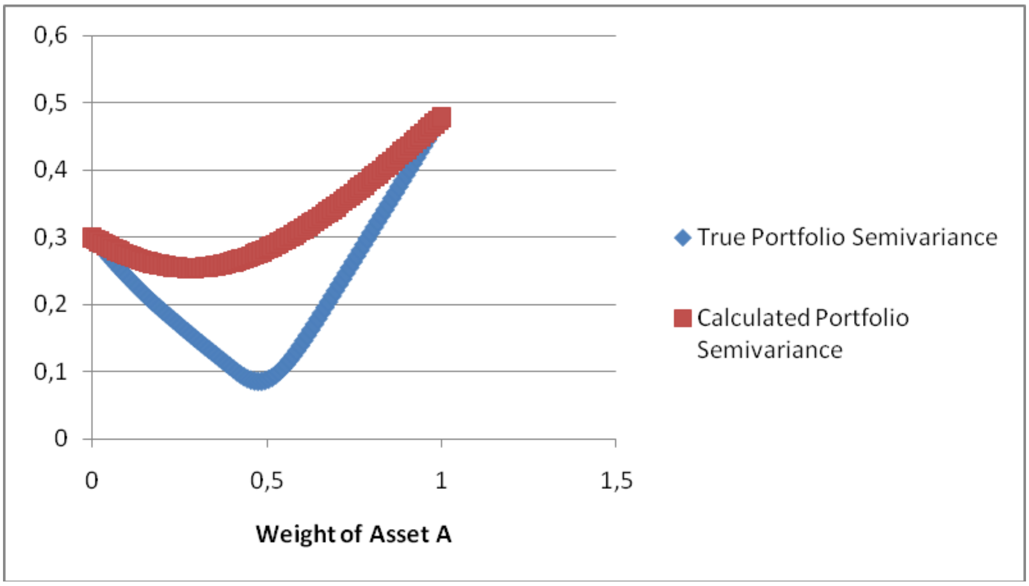

Figure 4. Sensitivity analysis of estimation (true and calculated values) under different weights of assets in a portfolio

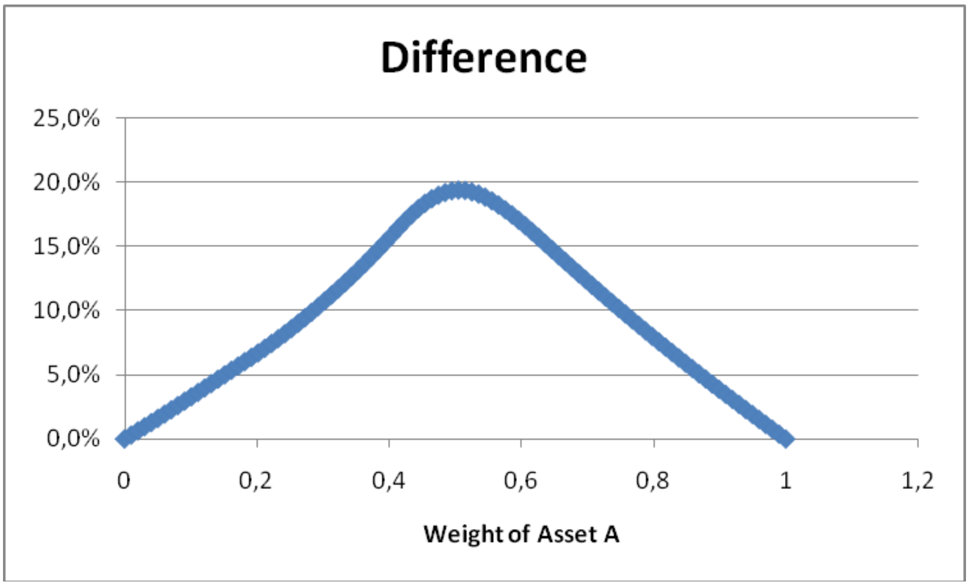

Figure 5. Sensitivity analysis of absolute estimation error under different weights of assets in a portfolio

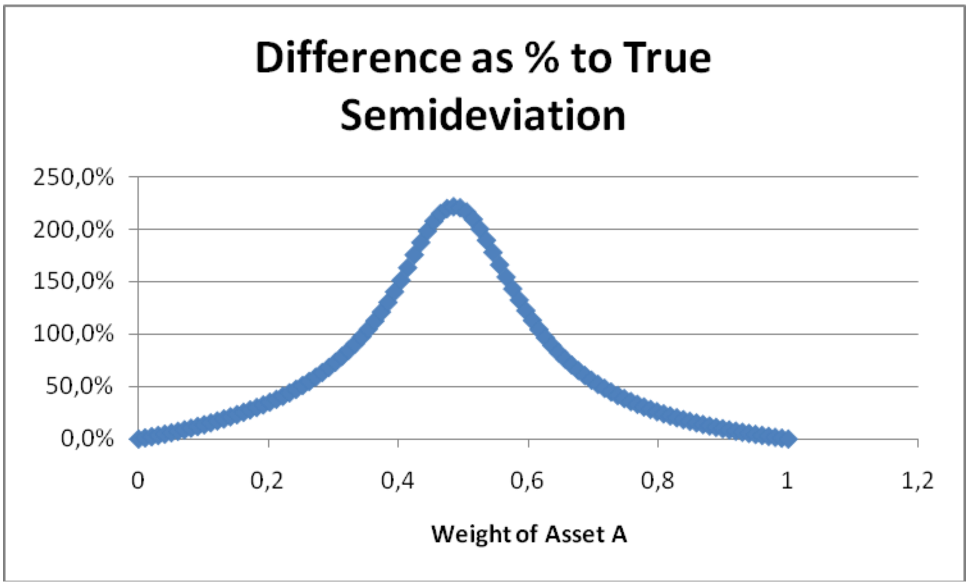

Figure 6. Sensitivity analysis of estimation percentage error under different weights of assets in a portfolio

The scenario 4, presented in Tables 7-8, illustrates the extreme case, when the assets have a correlation -1.

Table 7

Portfolio Semideviation Calculation – Fourth Scenario

Weights					
A	0.5				
B	0.5				
Year	Ra	Rb	Portfolio	min(rp-mu,0)	min(rp-mu,0)^2
1995	5.0%	87.3%	46.2%	0.0%	0,0000
1996	60.0%	32.3%	46.2%	0.0%	0,0000
1997	-5.0%	97.3%	46.2%	0.0%	0,0000
1998	93.3%	-1.0%	46.2%	0.0%	0,0000
1999	58.0%	34.3%	46.2%	0.0%	0,0000

2000	19.7%	72.6%	46.2%	0.0%	0,0000
2001	45.0%	47.3%	46.2%	0.0%	0,0000
2002	-10.0%	102.3%	46.2%	0.0%	0,0000
2003	59.0%	33.3%	46.2%	0.0%	0,0000
2004	78.0%	14.3%	46.2%	0.0%	0,0000
Average			**46.2%**	**0.0%**	**0,0%**
Semideviation					**0.0%**

Table 8

Estrada DCAPM framework measures calculation – Fourth Scenario

Year	Ra	Rb	min(ra-mu,0)	min(rb-mu,0)	min(ra-mu,0)^2	min(rb-mu,0)^2	min(ra-mu,0)*min(rb-mu,0)
1995	5.0%	87.3%	-35.3%	0.0%	0.1246	0.0000	0,0%
1996	60.0%	32.3%	0.0%	-19.7%	0.0000	0.0388	0,0%
1997	-5.0%	97.3%	-45.3%	0.0%	0.2052	0.0000	0,0%
1998	93.3%	-1.0%	0.0%	-53.0%	0.0000	0.2809	0,0%
1999	58.0%	34.3%	0.0%	-17.7%	0.0000	0.0313	0,0%
2000	19.7%	72.6%	-20.6%	0.0%	0.0424	0.0000	0,0%
2001	45.0%	47.3%	0.0%	-4.7%	0.0000	0.0022	0,0%
2002	-10.0%	102.3%	-50.3%	0.0%	0.2530	0.0000	0,0%
2003	59.0%	33.3%	0.0%	-18.7%	0.0000	0.0350	0,0%
2004	78.0%	14.3%	0.0%	-37.7%	0.0000	0.1421	0,0%
Average	**40.3%**	**52.0%**			**6.3%**	**5.3%**	
Semivariance					**25.0%**	**23.0%**	
Cosemivariance							**0.0000**
Downside Correlation							**0.00**
Downside Beta							**0.00**

The sensitivity analysis under scenario 4 is shown on figures 7-9. You can see that on figure 9 the percentage error of estimation can approach to infinity, when weight of asset A equal to 0.5. The true value of portfolio's semivarince is zero at this point, whereas the estimation according to DCAPM algorithms produces portfolio return semideviation of 17%. We can conclude that DCAPM is inappropriate for the situations when assets have negatively correlated returns, since as follows from mathematics of (2), downside correlation cannot be negative. In contrast the situation of negative correlation implies that upside returns to a great extent compensate downside returns. Although in practice, assets with negatively correlated returns are rare, they are still possible and this limitation of DCAPM should be taken into account.

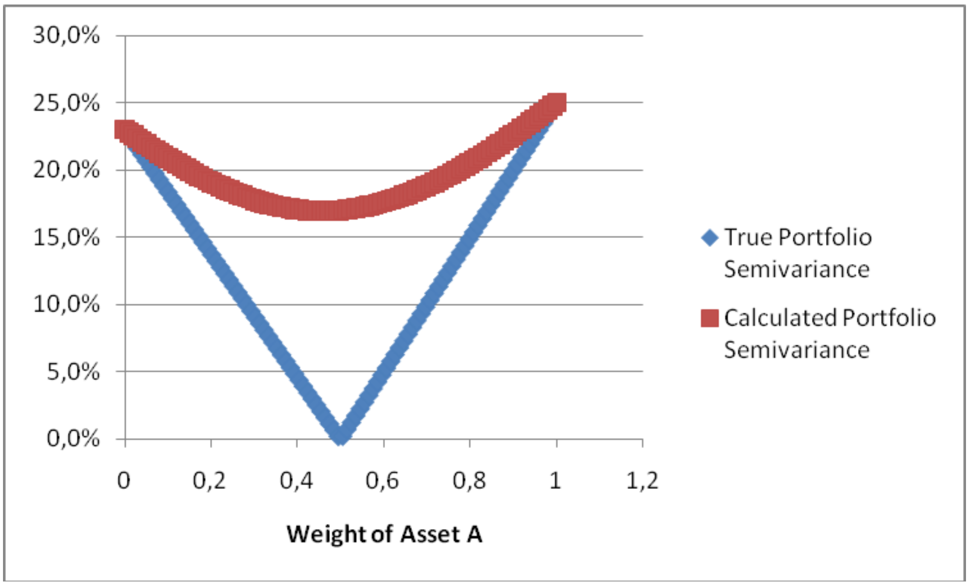

Figure 7. Sensitivity analysis of estimation (true and calculated values) under different weights of assets in a portfolio

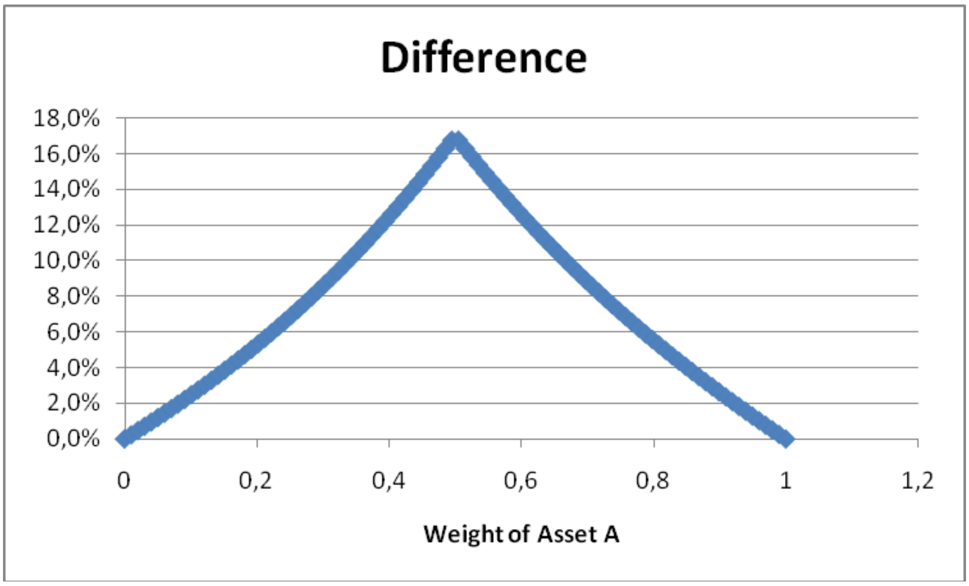

Figure 8. Sensitivity analysis of estimation error under different weights of assets in a portfolio

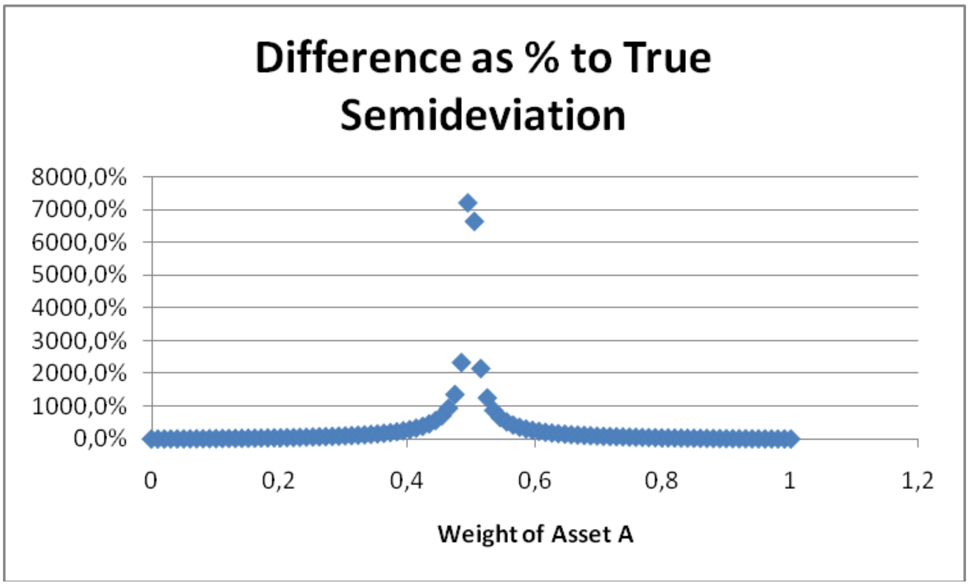

Figure 9. Sensitivity analysis of estimation percentage error under different weights of assets in a portfolio

The simulation shows that if assets are perfectly positively correlated the DCAPM's estimation percentage error is zero, and if the assets are perfectly negatively correlated the DCAPM's estimation percentage error can approach infinity. If the assets have zero correlation, the percentage error is quite significant (may reach 15% in the simulation under appropriate weights). The magnitude of the estimation error varies both with correlation between data, weights of the assets and the data themselves. So the algorithm cannot provide reliable estimates.

Estrada (2007) also states that his cosemivariance allows know only the approximation to the portfolio semivariance, but produces numbers that are very close to the exact figures. However he provides very dubious evidence to the accuracy of such approximation. He has used quite complex statistical exercise to test the accuracy of estimation based on a large sample. We have seen above the numbers may be very far from the exact numbers. The model doesn't stand the simple test. The efforts to adhere the CAPM to the downside deviation are plausible, but they encounter the unresolved problem of the determining the contribution of asset to the semideviation of the market portfolio. Approximation error will inevitably influence estimates of downside beta, for semideviation of the given asset, the market and cosemivariance between asset returns and market returns are integral part in the calculation of downside beta. The requirements for asset pricing models are much more severe than for portfolio optimization, as we need accurate estimations of marginal contribution of the given asset to the riskiness of market portfolio. Such estimations depend on accuracy of true dependency between assets' returns.

Also it is interesting to test DCAPM using ideal case of normally distributed assets. Substantiating his approach, Estrada (2000) refers to Bawa and Lindenberg (1977), who asserted that CAPM is a special case of Downside CAPM model, which is restricted to the symmetric and normal distribution of returns. DCAPM is a modification of classical CAPM model, which relaxes the assumption of symmetric and normally distributed returns. In other respects DCAPM relies on the same assumptions as classical CAPM model. DCAPM is obtained from CAPM by replacing variation with downside variation and by replacing correlation with downside correlation. DCAPM was designed to provide better estimates of beta than classic CAPM for cases of asymmetric distributions of returns. Therefore CAPM estimates of beta when assumption of normally distributed variables holds should be a standard for DCAPM model. For the case of normally distributed symmetric variables DCAPM should provide estimates, which are very close to standard beta. Below we will test this hypothesis by using several confirming and disconfirming instances. To do so we simulate two correlated distributions using Palisade @Risk software consisting of 1000 observations. Inputs are presented in Figures 10 and 11.

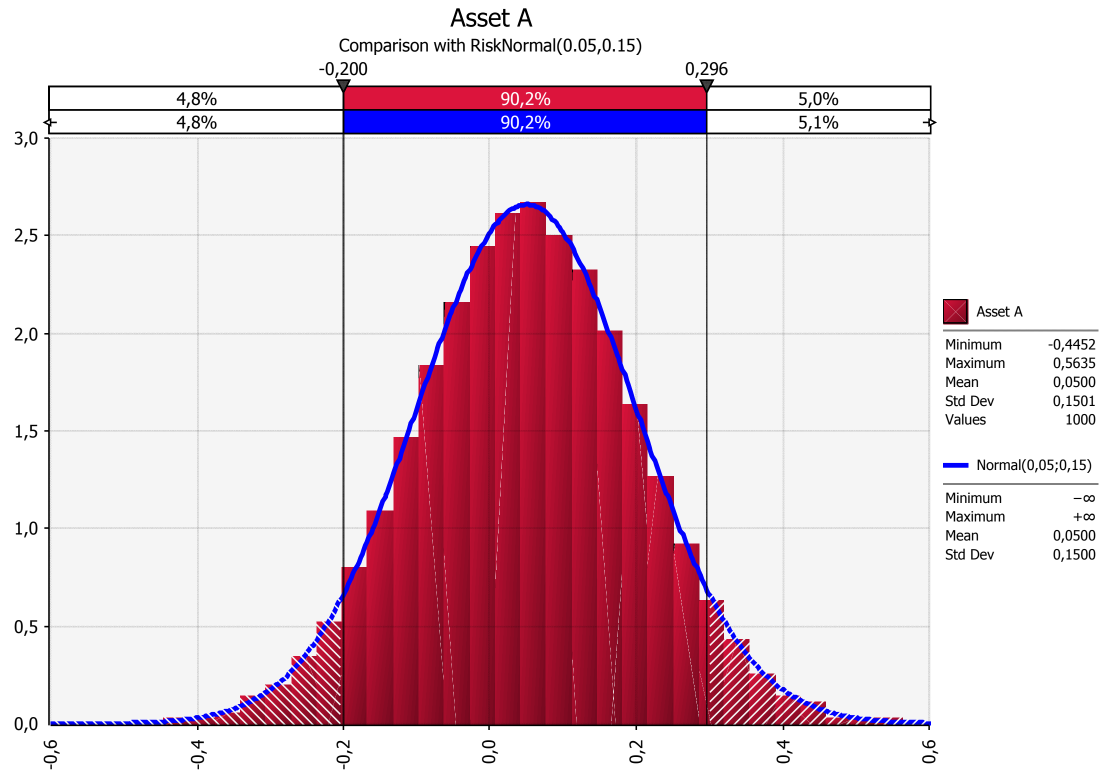

Figure 10. Distributions of returns of asset A

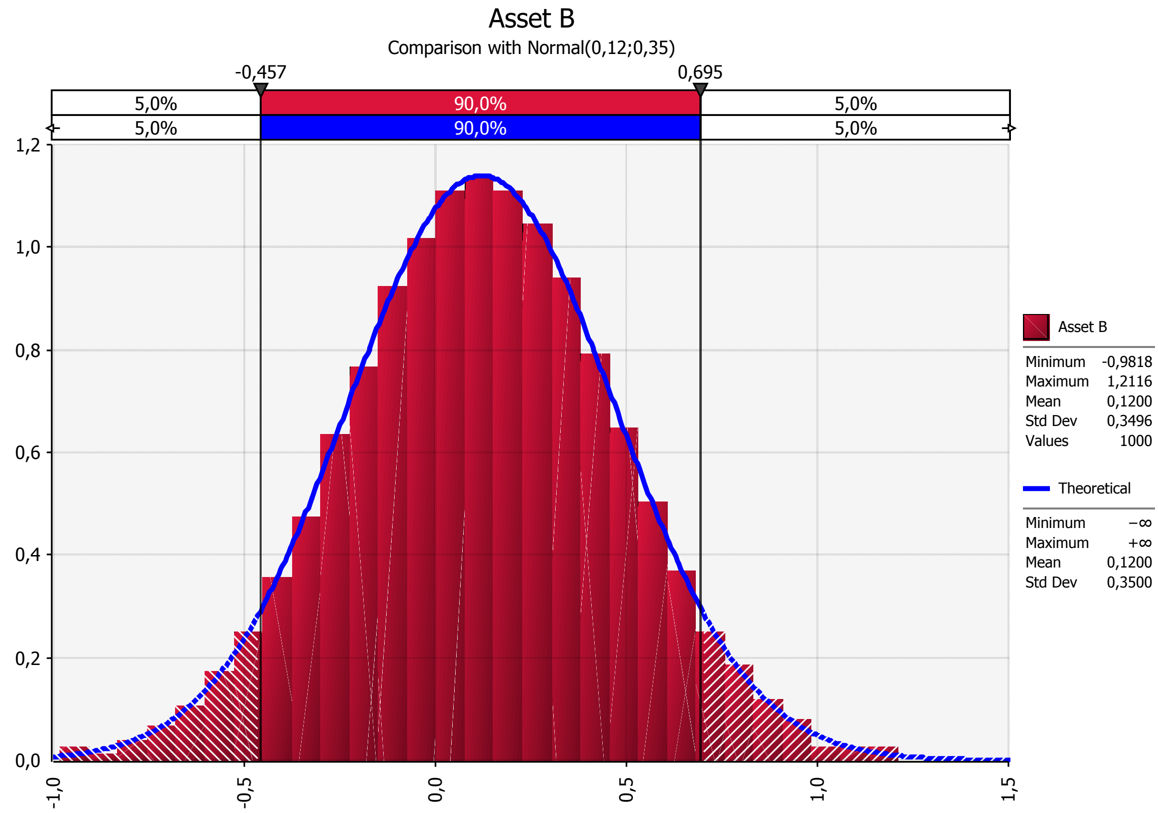

Figure 11. Distributions of returns of asset B

Simulated asset A has the following characteristics:
expected return – 4.99%;
sigma – 15.03%;
downside variance – 1.13%;
downside sigma – 10.64%.
Simulated asset B has the following characteristics:

expected return – 12%;
sigma – 34.96%;
downside variance – 6.10%;
downside sigma – 24.71%.

Correlation between returns of assets A and B is 0.3135, cosemivariance is 0.0140 and downside correlation is 0.5319. Let asset B be market returns. Then we can calculate traditional and downside beta for asset A. Surprisingly asset A traditional true beta is 0.1348, while downside beta is 0.229. As we can see DCAPM overestimates true beta for normally distributed asset returns by 69.9%. The security characteristic lines are presented in Figures 12 and 13.

Figure 12. Traditional security characteristic line for asset A

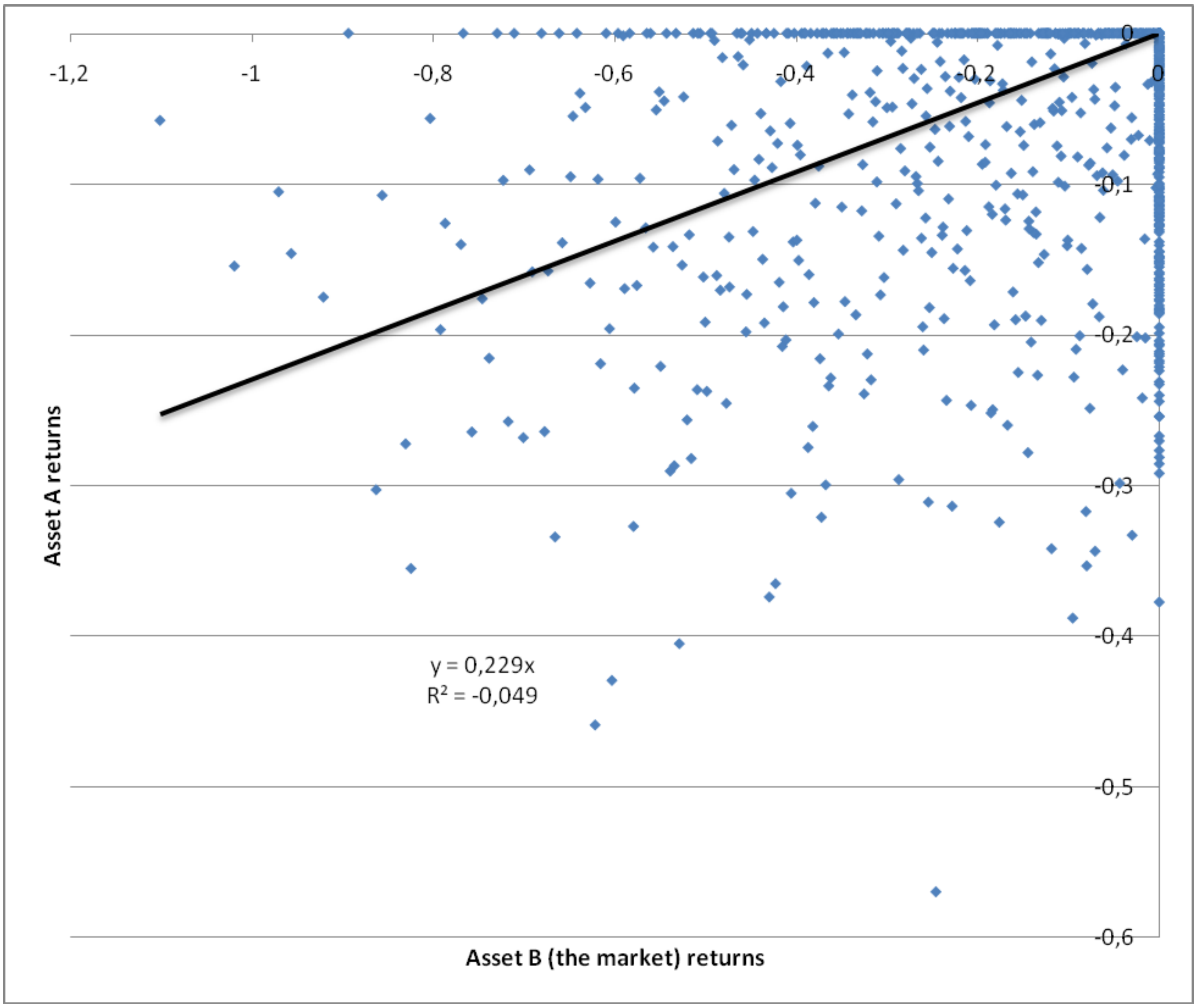

Figure 13. Downside security characteristic line for asset A

The difference between true traditional beta and downside is anticipated, because downside deviation is not compatible with standard deviation even for the case of perfectly normally distributed data. Therefore, even if we assume that the approximation error is not critical for our calculations, how should we calculate risk premium using downside beta? Let excess market return be 7%. Then according to traditional CAPM model risk premium for asset A is equal to 0.94% (0.1348*7%). But according to downside beta approach risk premium will be 1.6% (0.229*7%). The finding is that DCAPM can produce distorted estimates of beta and risk premium. The difference is because DCAPM approach ignores the ability of upside returns of asset A to hedge the downside returns of the market. And as we can see, this difference can be quite significant. Simulation shows that this difference depends on correlation between given asset and the market. If correlation is high, the difference between downside beta and true beta will be low. To test this conclusion we used the same inputs as shown above, but changed the correlation between assets A and B to 0.8. Under this value of correlation the true beta for asset A is 0.3424 and the estimate of downside beta according to DCAPM algorithm is 0.3527. The security characteristic lines are presented in Figures 14 and 15.

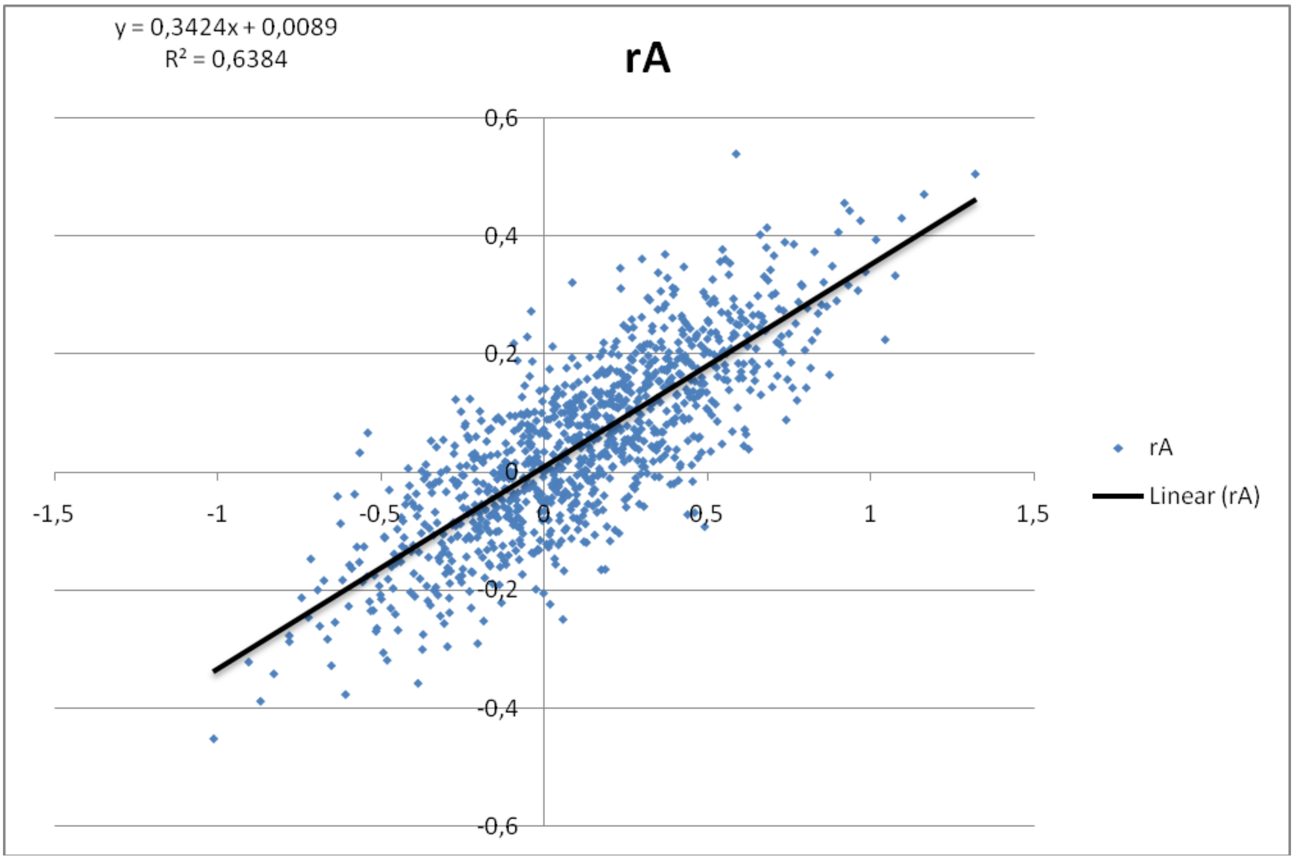

Figure 14. Traditional security characteristic line for asset A

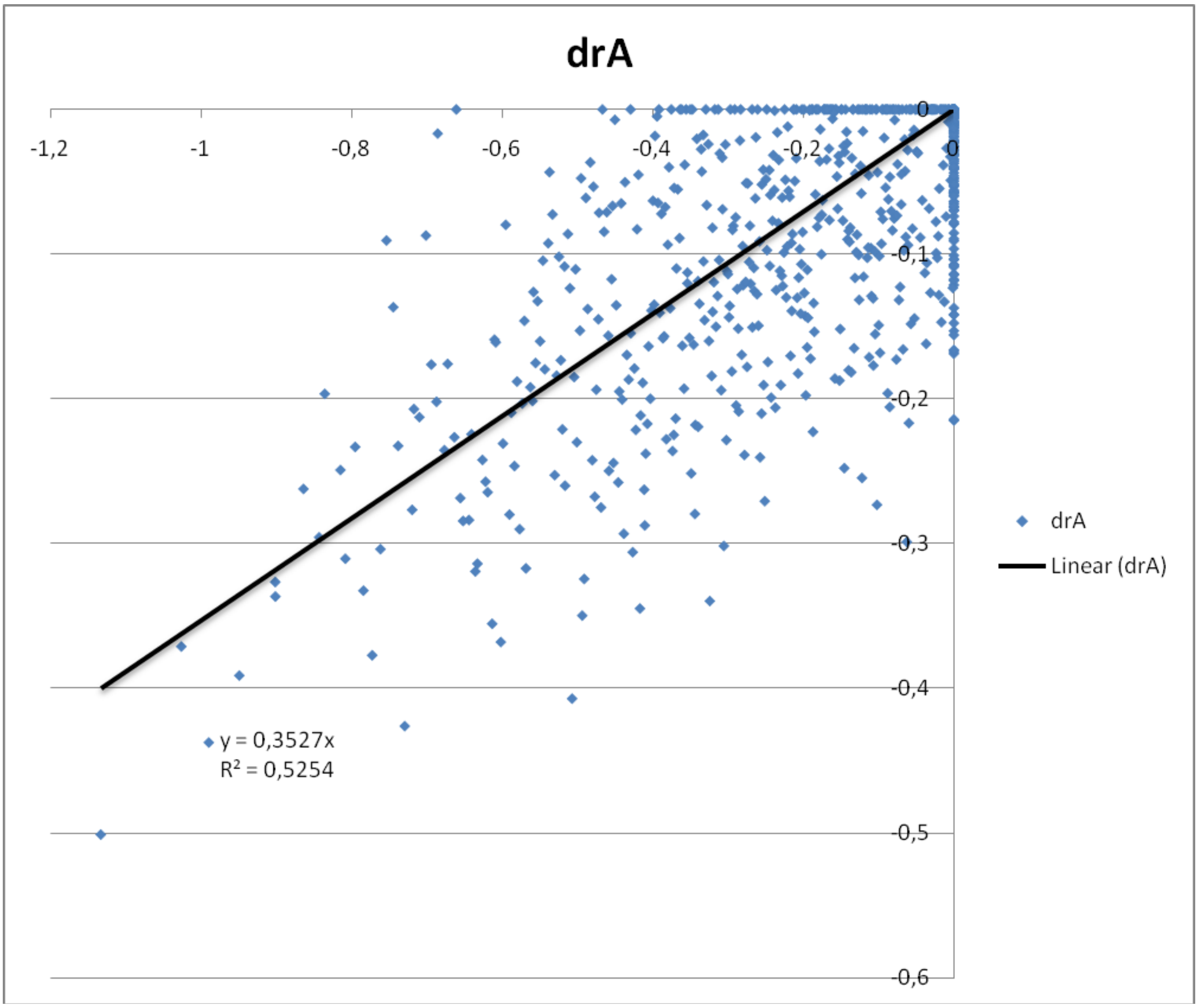

Figure 15. Downside security characteristic line for asset A

The simulations reveal that for highly correlated assets the estimated of beta according to DCAPM are very close to classical CAPM models. It is easy to explain, as for the assets with highly correlated returns the effect of compensation by upside returns of one asset the downside returns of another asset is weak enough. But if correlation is low, the difference between downside beta and true beta will be considerable. At medium correlation, the difference is also significant. Dependence of the approximation on correlation between assets makes the DCAPM model very questionable for the purposes of estimating betas and risk premiums of assets.

3. Conclusion

Current empirical and econometric research tends to explain the cross-section returns. The DCAPM support usually addresses to its better ability to explain cross-section returns on the imperfect and emerging markets, where the underlying distributions of returns do not hold normality and symmetry restrictions. However this argumentation is weak. The issue of the utmost importance is logical and mathematical correctness of the model.

The paper reveals that the cosemivariance (downside correlation) is a questionable measure and is not a reliable statistics. Downside portfolio return semivariance depends on the weights of the assets, correlation between those assets, asymmetry of their distributions, size of the sample and other factors. DCAPM algorithm provides good approximations for assets with highly correlated returns, but its accuracy significantly decreases for assets with lowly correlated returns. Also we revealed that DCAPM is very inaccurate for assets with negatively correlated returns, as the downside correlation statistic introduced by Estrada cannot be negative and completely ignore the possibility of upside returns of one asset to partly or completely cancel downside returns of another asset. The effect of compensation of upside returns of one asset the downside returns of another asset manifests itself especially in the situations of assets with lowly or negatively correlated returns. Simulations reveal that DCAPM is inaccurate for such situations even when assumption of normally distributed returns holds. Under such instability there is no meaningful interpretation of downside beta. The finding is that downside beta is inconsistent with traditional beta. This explains why empirical tests of DCAPM models are found to be controversial by researchers.

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