DOI: 10.17323/j.jcfr.2073-0438.15.1.2021.67-76
JEL Classification: D81, G31, O16, O22

Conceptual Problems in the Use of Risk-Adjusted Discount Rate for Risky Negative Cash Flows

Anastasia Blaset Kastro
Cand. Sci. (Econ.), Analyst, Center of Monitoring and Forecasting,
ORCID
E-mail: anblaset@gmail.com
All-Russian Research Institute of Metrology Service, Moscow, Russia

Nikolay Kulakov ✉
PhD in Engineering Science, Investment Analyst
ORCID
E-mail: nkulakov@gmail.com
The Moscow Investor Club, Moscow, Russia

Journal of Corporate Finance Research, Vol. 15, No. 1, pp. 67-76 (2021)

For citation: Kastro, A. B. и Kulakov, N. (2021) «Conceptual Problems in the Use of Risk-Adjusted Discount Rate for Risky Negative Cash Flows», Journal of Corporate Finance Research / Корпоративные Финансы | ISSN: 2073-0438, 15(1), cc. 67-76. doi: 10.17323/j.jcfr.2073-0438.15.1.2021.67-76.

Received 20 December 2020 | **Peer-reviewed** 10 January 2021 | **Accepted** 20 January 2021

Abstract

This paper examines the risk adjusted discount rate (RADR) method for evaluating risky nonconventional projects, which has been hotly debated over the last century [1]. Economists face the contradiction of using the NPV rule to evaluate projects with different levels of risk. According to the theory of investments, the higher the project risk, the greater the return for the investor. Therefore, an increased discount rate is used to evaluate a riskier project, as a result, the project's NPV decreases and the project is deemed less attractive or even unprofitable for investment. However, the NPV of a nonconventional investment project may increase through increasing the discount rate, and then the investor, following the NPV rule, will choose a riskier project out of two projects with the same yield. That does not correspond to the hypothesis about rational investor behavior.

We continue the study of the RADR method. Recently, published works [2–4] have proposed a solution to the debatable RADR problem. The GNPV method was used for evaluating risky nonconventional projects. We will evaluate these aspects of the recent literature. We examine the fallacy of the main arguments (to maintain value additivity and preclude arbitrage) justifying the application of a single rate to discount risky opposite sign cash flows. The future cash flows are estimated independently of the transactions preceding them, which seems illogical, so a risk penalty formula which adjusts the discount rate applied to risky negative cash flows is applied. The risk penalty is determined depending on the risk premium in the case of symmetric and asymmetric distribution of cash flow values.

Our results are applicable to a diverse range of business applications, including but not limited to well-known asset pricing models, short position analysis, determining fair insurance premiums, and calculating appropriate RADRs for public private partnerships.

Keywords: nonconventional projects, net present value (NPV), risk, risk adjustment discount rate (RADR), negative cash flows, risk premium, risk penalty

Introduction

The risk adjusted discount rate method (RADR) applied to stochastic negative cash flows when evaluating nonconventional projects was a matter of a serious argument over several decades [5]. According to investment theory, the higher the project risk, the more return, and to achieve this an investor is required. Therefore, in order to evaluate a riskier project, an increased discount rate is applied. As a result, the project NPV decreases, and the project is perceived as less attractive or even unfavorable for investment. However, the NPV of a nonconventional investment project may increase instead of decreasing along with discount rate growth. In such a case, an investor following the NPV rule will choose a riskier project out of two projects with equal profitability. Such a choice contradicts the hypothesis of rational investor behavior.

In relation to this, two positions were expressed regarding risk adjustment of discount rate for stochastic negative cash flows. The first position states that the RADR, applied to a future risky cash flow, is independent on whether the flow is positive or negative, and the RADR increases along with growth of cash flows risk. The second position states that the RADR applied to future stochastic cash flows of equal risk is different for positive and negative cash flows. The rate increases for positive cash flows and decreases for negative cash flows as they become riskier. In other words, supporters of the first position assert that the same rate should be applied to evaluation of opposite sign cash flows of equal risk. Supporters of the second position affirm that different rates should be applied to assess random positive and negative cash flows of equal risk.

The main arguments of the first position supporters are as follows: a single rate is necessary to preclude arbitrage [6; 7]; the NPV loses additivity at different rates [8; 9]; as risk grows a negative premium may approximate the RADR to −1, as a result, and the present value of negative cash flows will be infinite [10]. Their opponents attribute the difference in adjustment of discount rates to the different nature of opposite sign cash flows, and consequently to other risks of negative cash flows [11]; to an inverse correlation of negative cash flows and the market rate [12; 17]; and to different approaches to risk identification and mitigation: decrease of the expected benefit or increase of estimated costs [18; 19].

The problem of adjustment of the discount rate for stochastic negative cash flows has been unsolved for a long time. For a significant period of time no references were made to it, as if the problem did not exist. Consequently, there were controversial recommendations in the financial literature concerning adjustment of the discount rate applied to random negative cash flows. Managers had no idea when a positive or negative risk premium should be used and how it was calculated [20].

The contradiction is a result of a standard application of the methods developed to assess investments to evaluation of risky loans. Economists know very well that in order to evaluate loans one has to reverse the nonequality sign in the IRR rule intended to assess investments ($IRR > d$, where d is the discount rate), because for investments IRR is a return while for a loan, it represents an interest rate. “When we lend money, we want a high rate of return; when we borrow money, we want a low rate of return” [21]. Consequently, if IRR has different economic substance for investments and loans, it should be compared to different discount rates distinguished in economic substance. The NPV uses a single discount rate called ‘opportunity cost of capital’. In the evaluation of nonconventional projects, this rate is at the same time the rate of return and cost of capital. Thus, capital is lent and placed at the same rate, and this causes problems for the evaluation of nonconventional projects. Recently the generalized net present value (GNPV) method has been offered which uses two different rates to discount investments and loans which form a nonconventional project [22; 23]. The financial rate is used to attract funds for project financing while the reinvestment rate is used to invest them. Thereby, the GNPV method by default implies that in case of risk adjustment the financial rate is to be increased, while the reinvestment rate is to be reduced [2]. Thus, the GNPV method solves the problem of the RADR for evaluation of random negative cash flows.

This paper pursues several objectives: 1) to show the fallacy of old arguments justifying the same way of changing RADR in case of assessment of risky cash flows with opposite signs; 2) to define the risk penalty value on the basis of the risk premium, in order to change RADR relative to a risk-free rate when evaluating random negative cash flows; 3) to sort out the controversial recommendations offered in manuals concerning the RADR method in respect to assessment of risky nonconventional projects.

The paper has the following structure. The first section defines our identified problem and gives a brief review of the relevant background research in the field. The second section discusses old arguments justifying the single approach to adjustment of the discount rate applied to risky opposite sign cash flows - we will prove that these arguments are fallacious. In the third section, we derive a formula of risk penalty to define the RADR applied to stochastic negative cash flows in case of symmetric and asymmetric distribution of their values. We compared the values of risk penalty calculated by the obtained formula and presented in paper [2]. We presented a case describing use of the RADR method. Finally, in the conclusion, we summarize the main results.

Problem Statement

Let us remember the RADR problem which emerges in evaluation of nonconventional projects in uncertain environments. William Beedles [24] considered a nonconventional project with three cash flows: $ −5,000; $111,500 and $−6,600. Let us assume that the first and second cash flows are completely certain while the last one takes on a value of $−6,200 or $−7,000 with a probability of 50/50. According to the investment theory, uncertain cash flows should be discounted at the risk adjustment rate. Assuming such a rate is 9%, then the project NPV will be $–4.63. Let us suppose that in a similar project a random distribution of the third flow value of $–6,600 shows a greater dispersion being the mean of the two possibilities of $–5,200 and $–8,000 occurring with a probability of 50/50. This flow has higher risks, and therefore it should be discounted at an increased rate. If we take, for example, a rate of 11% the project NPV will be $+3.65. The result looks counterintuitive, because the project value should not grow as the risk increases. The remarkable thing is that this result did not strike Beedles as unusual. The role of the project NPV acquires more importance in the range of (0-15%) and achieves the maximum at the discount rate of 15%. He made the conclusion that the RADR method should not be used to assess nonconventional projects and offered to apply the certainty equivalent method (CE) in this case.

The CE method is considered to be an approach alternative to the RADR in the evaluation of risky investments. According to the CE uncertain expected cash flows are replaced with their certainty equivalents (guaranteed cash flows) and are discounted at a risk-free rate. Specialists consider the CE method to be more correct from the theoretical point of view than the RADR but the majority of companies apply the RADR more often [25; 26].

J. Miles and D. Choi [6], as supporters of the single approach to RADR adjustment, strongly criticized Beedles' offer to apply the CE method pointing out that it did not conform to the value additivity principle and arbitrage probability. R. Ariel [7] expanded on their arguments concerning the RADR method. However, there is an error in their reasoning which will be discussed in the following section.

According to the hypothesis of a rational investor who avoids risk, the project value should decrease as uncertainty increases. Therefore, performing risk adjustment, the discount rate should be increased for inflows, and reduced for outflows relative to a risk-free rate. A lot of economists hold to this view [11–19]. However, a decrease of the rate in case of riskier negative cash flows contradicts the investment theory (greater risk requires greater return). In order to eliminate this contradiction, it was proposed to evaluate the risk level of project cash flows separately and depending on the risk level, and use different rates. For example, M. Ehrhardt and P. Daves [11] offered to consider cash costs at the end of a nonconventional project as nonoperating flows of another nature and smaller risks. However, first, the offer to consider closure costs as non-operating and, consequently less risky, is not totally correct [2]. Second, the adjusted rate will anyway be risk-free to a greater extent (because the risk premium is positive), hence, the risk outflows cost will exceed similar risk-free outflows.

The economists who assessed protection against environmental risks emphasized that the risk premium sign for discount rate adjustment relative to a risk-free rate depends on the way of evaluation of the cost of insurance of environmental risk consequences: decrease of the expected benefit or increase of estimated costs [18; 19]. Insurance companies demand a greater premium in order to cover more uncertain cases in future and for this reason they reduce the rate more in order to assess the current value of future more uncertain payments [27].

This review shows that in order to perform different adjustments relative to a risk-free rate, the discount rates for positive and negative cash flows should be different. Recently there appeared publications which proved that nonconventional projects should be evaluated by the Generalized net present value (GNPV) method which by definition applies different rates to discount investments and loans forming such projects [22; 23]. The funds are attracted at the financial rate but invested at the reinvestment rate. The GNPV method by default implies an increase of the financial rate and decrease of the reinvestment rate when performing risk adjustment [2].

Depreciation and Arbitrage When Using Different Rates

J. Miles and D. Choi [6], R. Berry and R. Dyson [8] followed by R. Ariel [7] came up with valid arguments justifying use of the single rate (more specifically, a single method of risk adjustment of a risk-free rate) for discounting of random opposite sign cash flows. Probably, these old arguments are an obstruction to the final solving of the problem of the RADR applied to risky negative cash flows.

J. Miles and D. Choi [6] strongly criticized Beedles' offer to apply the CE method for assessment of random nonconventional projects. Let us quote a translation: "Assume company A has to make an uncertain payment of X US dollars to company B at the end of the current period. If company A uses $\alpha_A > 1$ as the CE factor to assess a negative cash flow the cost of this outflow will be calculated as follows:

$$V_A[X] = \frac{\alpha_A \cdot X}{1 + r_f}, \qquad (1)$$

where r_f – a risk-free rate; X – expected cash flow; α_A – CE factor which forms a guaranteed (risk-free) outflow $\alpha_A > 1$".

According to Beedles, company B evaluates such uncertain cash inflow using $\alpha_B < 1$ as the CE factor:

$$V_B[X] = \frac{\alpha_B \cdot X}{1 + r_f}, \qquad (2)$$

As long as $\alpha_A > \alpha_B$, company A assesses the payment value higher than company B i.e., $V_A[X] > V_B[X]$.

Then Miles and Choi reason as follows: "In perfect markets this difference in the value results in profit due to arbitrage because the same asset X is evaluated by the market players in a different way. So, a rational investor will offer company A to make payment X to company B at the price of $V_A[X]$. At the same time, he will offer company B the amount of $V_B[X]$ in exchange to a promised future payment of X. The arbitrage profit of $V_A[X] - V_B[X]$ will make market players compete reducing $V_A[X]$ and increasing $V_B[X]$ till $V_A[X] = V_B[X]$".

On the basis of the condition of equality of present values, to preclude arbitrage they made the conclusion that the rates should be equal, otherwise the law of conservation of value is not complied with. Therefore, in perfect markets evaluation of cash inflows and outflows should be identical.

The error in Miles and Choi's reasoning consists in the fact that they identify the present value of a future payment X from company A to company B with a certain amount which company B pays and company A receives at present. In fact, when a deal is concluded at the beginning of the period the asset value equals some value of Y which in a perfect market is defined irrespective of the players[1]. For this reason, company A evaluates a loan (Y; $-X$) while company B evaluates an investment ($-Y$; X). In order to preclude arbitrage in perfect markets it is necessary to even the present values of an investment and loan for companies B and A and in no way – the present values of individual random cash flows X and $-X$. Assuming that $X > 0$ and $Y > 0$ we will define the present value of a loan for company A by means of the CE method:

$$PV_A = Y - \frac{\alpha_A \cdot X}{1+r_f}. \quad (3)$$

For company B the net present value of an investment equals

$$PV_B = -Y + \frac{\alpha_B \cdot X}{1+r_f}. \quad (4)$$

The condition of arbitrage preclusion in a perfect market $PV_A = PV_B$ will be fulfilled if

$$2Y = \frac{(\alpha_A + \alpha_B)\cdot X}{1+r_f}.$$

If we assume that a random cash flow X has symmetric distribution then $\alpha_A = 1+\alpha$, $\alpha_B = 1-\alpha$, where $0 < \alpha < 1$.

The result is $Y = \dfrac{X}{1+r_f}$.

Thus, the current payment value equals the present value of the expected future payment calculated at a risk-free rate. Therefore, arbitrage in such a deal is impossible.

Ariel makes a similar error in his reasoning when assessing long and short positions of a risky asset. He also identifies the present value of a future random cash flow to the current value at which the asset is traded. The error is caused by the fact that future flows are evaluated irrespective of the transactions which generate them. First, an investor has to buy an asset and then sell it and vice versa.

Evaluation of Stochastic Investments and Loans

Paper [2] showed that discount rates of positive and negative present values differ in their nature and offered a RADR calculation method for investments and loans numerically. In this paper we will reproduce this method from an analytical point of view.

Symmetric Distribution of Cash Flow Random Values

Assume the first cash flow is precisely known and equals CF_1 while the second cash flow is a random value described by the normal law of distribution with the mean of $<CF_2>$ and a root-mean square error (RMSE) σ.

The theory of probability showed that if some normally distributed random variable x has the mean value of M and a root-mean-square deviation of σ, the probability of its getting into the interval of $x < y$ is predetermined by the probability integral of $F(x)$ [28]

$$P(x<y) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{y} e^{-\frac{(x-M)^2}{2\sigma^2}} = $$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{y-M}{\sigma}} e^{-\frac{t^2}{2}} = F\left(\frac{y-M}{\sigma}\right) . \quad (5)$$

The probability of profit CF_2 is less than *the* CE_i value and equals

$$P(CF_2 < CE_i) = F\left(\frac{CF_i - \langle CF_2\rangle}{\sigma}\right)$$

Suppose the profit CE_i is a certainty equivalent in case of investments. For the random variable CF_2 with the normal law of distribution the probability of profit is anyway not equal to zero and less than CE_i. If we let this probability be equal to δ, then the certainty equivalent for investment may be determined by the following formula:

$$F\left(\frac{CF_i - \langle CF_2\rangle}{\sigma}\right) = \delta \text{ , hence } CE_i = \langle CF_2\rangle + \sigma F^{-1}(\delta)$$

where the inverse function is $F^{-1}(\delta) < 0$.

The present values of cash flows calculated applying the CE and RADR methods should be equal, so we have

$$\frac{CE_i}{1+r_f} = \frac{\langle CF_2\rangle + \sigma F^{-1}(\delta)}{1+r_f} = \frac{\langle CF_2\rangle}{1+r_{RADR}} = \frac{\langle CF_2\rangle}{1+r_f+RP}, \quad (6)$$

where r_f – risk-free rate; RP – risk premium.

[1] Miles and Choi's reasoning may be considered as a mechanism for establishing an equilibrium price of a deal.

$$\frac{\langle CF_2\rangle+\sigma F^{-1}(\delta)}{1+r_f}=\frac{\langle CF_2\rangle}{1+r_f+RP}\Rightarrow(1+r_f)\langle CF_2\rangle-(1+r_f+RP)\big(\langle CF_2\rangle+\sigma F^{-1}(\delta)\big)\Rightarrow$$

$$\Rightarrow(1+r_f)\sigma F^{-1}(\delta)+RP\big(\langle CF_2\rangle+\sigma F^{-1}(\delta)\big)=0\Rightarrow RP=\frac{(1+r_f)\sigma F^{-1}(\delta)}{\langle CF_2\rangle+\sigma F^{-1}(\delta)}. \qquad (7)$$

Now let us consider a loan. The second cash flow in case of a loan is also a random variable distributed according to the normal law, but with the mean value of – $<CF_2>$ and a root-mean-square deviation of σ. As in case with investments we predetermine the probability of undesirable outcomes as δ, and define the minimum allowed outflow using the probability integral formula. Suppose this minimum allowed outflow is the certainty equivalent CE_b for the loan:

$$F\left(\frac{CF_b+\langle CF_2\rangle}{\sigma}\right)=\delta\text{, hence } CF_b=\langle CF_2\rangle-\sigma F^{-1}(\delta)\text{,}$$

$$-\frac{CE_b}{1+r_f}=\frac{-\langle CF_2\rangle+\sigma F^{-1}(\delta)}{1+r_f}=\frac{-\langle CF_2\rangle}{1+p_{RADR}}=\frac{-\langle CF_2\rangle}{1+r_f+RP^*}\text{,}$$

where RP^* – risk penalty.

$$-\langle CF_2\rangle(1+r_f)=\big(-\langle CF_2\rangle+\sigma F^{-1}(\delta)\big)(1+r_f+RP^*)\Rightarrow$$

$$(1+r_f+RP^*)\sigma F^{-1}(\delta)=\langle CF_2\rangle RP^*\Rightarrow RP^*=$$

$$=\frac{(1+r_f)\sigma F^{-1}(\delta)}{\langle CF_2\rangle-\sigma F^{-1}(\delta)}. \qquad (8)$$

The relation of inverse values of risk premium and penalty may be presented as follows

$$\frac{1}{RP}=-\frac{\langle CF_2\rangle+\sigma F^{-1}(\delta)}{(1+r_f)\sigma F^{-1}(\delta)}\text{ and }\frac{1}{RP^*}=\frac{\langle CF_2\rangle-\sigma F^{-1}(\delta)}{(1+r_f)\sigma F^{-1}(\delta)}.$$

The sum of these inverse values is:

$$\frac{1}{RP}+\frac{1}{RP^*}=-\frac{\langle CF_2\rangle+\sigma F^{-1}(\delta)}{(1+r_f)\sigma F^{-1}(\delta)}+$$

$$+\frac{\langle CF_2\rangle-\sigma F^{-1}(\delta)}{(1+r_f)\sigma F^{-1}(\delta)}=\frac{-2\sigma F^{-1}(\delta)}{(1+r_f)\sigma F^{-1}(\delta)}=$$

$$=-\frac{2}{1+r_f}. \qquad (9)$$

After several more transformations we obtain:

$$\frac{1}{RP}+\frac{1}{RP^*}=-\frac{2}{1+r_f}\Rightarrow\frac{1}{RP}+\frac{1}{1+r_f}=$$

$$=-\left(\frac{1}{RP^*}+\frac{1}{1+r_f}\right)\Rightarrow\frac{1+r_f+RP}{RP(1+r_f)}=$$

$$=-\frac{1+r_f+RP^*}{RP^*(1+r_f)}\Rightarrow\frac{1+r_f+RP}{RP}=$$

$$=-\frac{1+r_f+RP^*}{RP^*}\Rightarrow\frac{RP}{1+r_f+RP}=$$

$$=-\frac{RP^*}{1+r_f+RP^*} \qquad (10)$$

Formula (10) defines the relation between the risk premium and penalty for any symmetric distribution of the random variable. The left-hand side defines the present value of the risk premium calculated at the RADR rate applied to inflows. The right-hand side is the present value of the risk penalty calculated at the RADR rate applied to outflows. Relation (10) is universal. Its economic substance will be explained below.

After simple transformations of equation (10) we have a formula to calculate the risk penalty.

$$RP(1+r_f+RP^*)=-RP^*(1+r_f+RP)\Rightarrow$$

$$RP+RP\cdot r_f+RP\cdot RP^*=-RP^*-RP^*\cdot r_f+RP\cdot RP^*\Rightarrow$$

$$RP^*(1+r_f+2RP)=-RP(1+r_f)\Rightarrow$$

$$RP^*=-\frac{RP}{1+2RP/(1+r_f)}. \qquad (11)$$

Consequently, RADR for evaluation of risky outflows will be

$$p_{RADR}=r_f-\frac{RP}{1+2RP/(1+r_f)}. \qquad (12)$$

Paper [2] obtained target values of risk premium for risk adjustment of the rates applied respectively to assess investments and loans in case of normal distribution of random cash flows. See these values in Table 1.

Table 1. The risk premium and penalty depending on the level of risk (%)

σ, $	r_f	Premium	Penalty
100	30	16.0	–12.8
200	30	36.5	-23.4

Let us employ formula (11) to calculate the risk penalty depending on risk premium values and the risk-free rate. For a smaller risk level σ = $ 100 we obtain:

$$RP^*=-\frac{16\%}{1+32\%/1.3}=-\frac{16\%}{1.246}=-12.8\%$$

In case of a high-risk level σ = $ 200 we have

$$RP^* = -\frac{36.5\%}{1+73\%/1.3} = -\frac{36.5\%}{1.562} = -23.4\%.$$

As we see, the obtained values of the risk penalty are the same as the values in the table.

Although formula (11) defining the risk penalty was obtained for random flows distributed according to the normal law, it is true for any symmetric distribution.

Indeed, as long as certainty equivalents of the random inflow and outflow are equal, consequently $CE_i = \langle CF_2 \rangle + \sigma F^{-1}(\delta)$ and $CE_b = \langle CF_2 \rangle - \sigma F^{-1}(\delta)$ the CE factor will be as follows

$$\alpha = \frac{\sigma F^{-1}(\delta)}{\langle CF_2 \rangle}.$$

Therefore, for deriving the formula which defines the risk penalty, the value of the CE factor is of no importance while certainty equivalents of inflow and outflow have to be symmetric with respect to the mean value.

$$CE_i = \langle CF_2 \rangle (1+\alpha), \; CE_b = \langle CF_2 \rangle (1-\alpha). \quad (13)$$

Asymmetric Distribution of Cash Flow Random Values

Now, we consider the case when random values distribution of a cash flow is not symmetric. Assume, for example

$$CE_i = a \cdot \langle CF \rangle, \; CE_b = \frac{1}{a} \langle CF \rangle. \quad (14)$$

Then on the basis of the equation of present values of a future inflow calculated by means of the RADR and CE methods we have

$$\frac{CE_i}{1+r_f} = \frac{a \cdot \langle CF_2 \rangle}{1+r_f} = \frac{\langle CF_2 \rangle}{1+r_{RADR}} \Rightarrow a = \frac{1+r_f}{1+r_{RADR}}. \quad (15)$$

Relation (15) was obtained by A.A. Robichek and S.C. Myers as a necessary and sufficient condition of equivalence of the CE and RADR methods applied to evaluate random cash flows [29].

On the basis of the equation of present values of a future outflow calculated by means of the RADR and CE methods, we have:

$$\frac{CE_b}{1+r_f} = \frac{1}{a} \cdot \frac{\langle CF_2 \rangle}{1+r_f} = \frac{\langle CF_2 \rangle}{1+p_{RADR}} \Rightarrow a = \frac{1+p_{RADR}}{1+r_f}. \quad (16)$$

Making (15) and (16) equal and making the change of, $r_{RADR} = r_f + RP$,we have

$$\frac{1+p_{RADR}}{1+r_f} = \frac{1+r_f}{1+r_f+RP} \Rightarrow p_{RADR} = \frac{(1+r_f)^2}{1+r_f+RP} - 1 =$$

$$= \frac{1+2r_f+r_f^2-1-r_f-RP}{1+r_f+RP} \Rightarrow p_{RADR} =$$

$$= \frac{r_f(1+r_f)-RP}{1+r_f+RP} = \frac{r_f(1+r_f+RP)-r_f \cdot RP - RP}{1+r_f+RP} =$$

$$= r_f - \frac{(1+r_f) \cdot RP}{1+r_f+RP} \Rightarrow p_{RADR} = r_f - \frac{RP}{1+RP/(1+r_f)}. \quad (17)$$

$$RP^* = -\frac{RP}{1+RP/(1+r_f)}. \quad (18)$$

Relation (18) defines the risk penalty value for random cash outflows with asymmetric distribution.

Y. Gallagher and J. Zumwalt [10] pointed out that as the rate of p_{RADR} approximates –1 the value of negative cash flows increases infinitely. Is it possible in real life? As the risk level of cash flows increases, the risk premium *RP* grows. But, even if the risk premium increases infinitely, the risk penalty for symmetric distribution is limited to the value of:

$$RP^* = -\lim_{RP\to\infty} \frac{RP}{1+2RP/(1+r_f)} =$$

$$= -\lim_{RP\to\infty} \frac{RP(1+r_f)}{1+r_f+2RP} = -0.5(1+r_f).$$

Formula (12) also implies that if $p_{RADR} = -1$ the r_{RADR} rate will also be –1:

$$p_{RADR} = r_f - \frac{RP}{1+2RP/(1+r_f)} = -1 \Rightarrow r_f + 1 =$$

$$= \frac{RP(1+r_f)}{1+r_f+2RP} \Rightarrow 1+r_f+2RP = RP \Rightarrow r_f +$$

$$+RP = -1 \Rightarrow r_{RADR} = -1.$$

Consequently, the expected return on investment and the expected interest rate are -100%. This means that the investor expects to lose all invested funds and the borrower is not going to repay the debt in future. There is no contradiction in this reasoning and if it is not a fraudulent deal, a rational investor is unlikely to conclude it.

For asymmetric distribution of random flows, as the risk premium grows the risk penalty value tends to $RP^* = -(1+r_f)$ and consequently, $p_{RADR} = r_f + RP^* = -1$.

However, it is possible when $RP >> 1+r_f$ i.e., if the premium exceeds 100% by far. On the other hand, if $p_{RADR} = -1$, the risk-free rate r_f equals –1.

Practical Implementation of the RADR Method

Let us suppose that we have to assess a nonconventional project with cash flows from Table 2 [22]. All cash flows, except for the initial one, are random variables with mean values as in Table 2.

Table 2. Evaluating a nonconventional project ($)

Project	CF_0	CF_1	CF_2	CF_3	r_f, %	r_{RADR}, %	p_{RADR}, %	*GNPV*
A	–100	75	150	–100	10	17	3.8	3.3
A'	–100	75	150	–100	10	20	1.5	–1.7

The risk level of the project cash flows is offset by the risk premium which changes in the range of 7–10%. The capital cost for the company which decides to participate in the project is 10%. It is the rate at which the company may attract and lend funds implementing its projects without risk.

If we evaluate the project as an investment, at the r_{RADR} within 17–20% according to the NPV rule the project should be accepted because

$$NPV(17\%) = 11.2 > 0;\ NPV(20\%) = 8.2 > 0\ .$$

However, this conclusion is wrong. The last cash flow of the project is negative and the RADR rate intended for positive cash flows cannot be applied to it. We will use the GNPV method to assess this project. Table 2 states p_{RADR} values calculated by formula (12) for random negative cash flows with symmetric distribution. Let us calculate the GNPV

$$GNPV(17\%, 3.8\%) = 3.3 > 0;$$
$$GNPV(20\%, 1.5\%) = -1.7 < 0\ .$$

As we see, as cash flows become riskier the present value of the project decreases and the project is perceived as more attractive. As long as change of the risk level within the expected range renders the project ineffective (GNPV < 0) it should be rejected.

Conclusion

Recently there were serious debates concerning the approach to risk adjustment of the discount rate in the NPV method when evaluating nonconventional (combined) projects in an uncertain environment. Some scientists presumed that project profit and costs with an equal risk level should be discounted at the same rate according to the risk-return ratio. Others thought that the risk premium for positive and negative cash flows with an equal risk level should be different.

A recently published paper [2] showed that the same approach to risk adjustment of the discount rate applied to random opposite sign cash flows stems from imperfection of the NPV method which use for evaluation of nonconventional projects is not always correct. The NPV method applies the same discount rate (opportunity cost of capital) to assess investments and loans which form a nonconventional project. This rate serves both as the required return for investment with a similar risk level and as cost of capital used for investment funding. However, it is commonly known that the IRR rule has opposite signs when investment and loans are evaluated because IRR itself has different sense for investment and loans. Therefore, the discount rates should be different.

In this paper we have eliminated the root causes of the problem of the RADR applied to random negative cash flows. We have considered the reasoning justifying application of the same risk adjusted discount rate to evaluate random cash flows of opposite signs. In the opinion of the scientists who think that only the RADR should be applied, when different rates are used to assess cash inflows and outflows there arises "arbitrage probability" and "depreciation". These arguments are based on a false conclusion that the present value of a cash flow for purchase or sale of an asset in future equals the price at which the asset is traded now. Therefore, if participants use different rates to assess the asset value it will cause "arbitrage probability" and "depreciation". In fact, the price at which the asset is traded in a perfect market is defined irrespective of the players' expectations. It is also shown that expectations related to evaluation of the asset price in future depend on the operations performed with the asset now (purchase or sale). A deal comprised by two cash flows (long and short sales) instead of just a future cash flow should be evaluated. In order to assess these differently directed deals different rates should be applied, therefore no "arbitrage probability" and "depreciation" takes place.

On the basis of the condition of equality of present values of a short and long sale of the asset the RADR formula was derived to evaluate uncertain negative cash flows. The risk penalty is defined depending on a risk-free rate and risk premium for symmetric and asymmetric distribution of random cash flows.

The offered approach accords the controversial recommendations which one can still find in textbooks concerning risk adjustment of the discount rate applied to assess the value of future random cash flows. It provides a possibility to apply investment evaluation methods under risk and uncertainty to evaluation of risky loan projects and nonconventional projects in accordance with theory.

Our proposed approach may be applied to expand existing asset pricing models in order to evaluate a short position, calculate a fair insurance premium, define an appropriate rate for the assessment of public-private partnership projects' value, and other business applications.

References

1. Blaset Kastro A.N., Kulakov N.Yu. Definition of the concepts of conventional and nonconventional projects. *Biznes-informatika = Business Informatics*. 2016;(2):16-23. DOI: 10.17323/1998-0663.2016.2.16.23
2. Blaset Kastro A., Kulakov N. An application of the RADR method for risky cash outflows. *Korporativnye finansy = Journal of Corporate Finance Research*. 2018;12(4):61-70. (In Russ). DOI: 10.17323/j.jcfr.2073-0438.12.4.2018.61-70
3. Blaset Kastro A., Kulakov N. Adjusting the discount rate for assessing risky cash outflows. In: Systems analysis in economics – 2018. Proc. 5th Int. sci.-pract. conf.-biennale (Nov. 21-23, 2018). Moscow: Prometei; 2018:233-236. (In Russ.). DOI: 10.33278/SAE-2018.rus.233-236
4. Blaset Kastro A., Kulakov N. Risk-adjusted discount rates and the present value of risky nonconventional projects. *The Engineering Economist*. 2020. DOI: 10.1080/0013791X.2020.1815918
5. Cheremushkin S. The proper way to treat risky cash outflows in evaluation and capital budgeting: Positive or negative risk premium? *Finansy i kredit = Finance and Credit*. 2009;15(28):36-51. (In Russ).
6. Miles J., Choi D. Comment: Evaluating negative benefits. *Journal of Financial and Quantitative Analysis*. 1979;14(5):1095-1099. DOI: 10.2307/2330311
7. Ariel R. Risk adjusted discount rates and the present value of risky costs. *The Financial Review*. 1998;33(1):17-30. DOI: 10.1111/j.1540-6288.1998.tb01604.x
8. Berry R., Dyson R. On the negative risk premium for risk adjusted discount rates. *Journal of Business Finance and Accounting*. 1980;7(3):427-436. DOI: 10.1111/j.1468-5957.1980.tb00211.x
9. Berry R., Dyson R. On the negative risk premium for risk adjusted discount rates: A reply. *Journal of Business Finance and Accounting*. 1983;10(1):157-159. DOI: 10.1111/j.1468-5957.1983.tb00419.x
10. Gallagher T., Zumwalt J. Risk-adjusted discount rates revisited. *The Financial Review*. 1991;26(1):105-114. DOI: 10.1111/j.1540-6288.1991.tb00372.x
11. Ehrhardt M., Daves P. Capital budgeting: The valuation of unusual, irregular, or extraordinary cash flows. *Financial Practice and Education*. 2000;10(2):106-114.
12. Sweetser A.G., Roden P.F., S. Cooper S.K., Strawser R.H., Pettway R.H., Celec S.E. Thrust and parry. *Financial Management*. 1975;4(4):7-11. DOI: 10.2307/3664935
13. Celec S.E., Pettway R.H. Some observations on risk-adjusted discount rates: A comment. *The Journal of Finance*. 1979;34(4):1061-1063. DOI: 10.2307/2327072
14. Everett J.E., Schwab B. On the proper adjustment for risk through discount rates in a mean-variance framework. *Financial Management*. 1979;8(2):61-65. DOI: 10.2307/3665351
15. Booth L.D. Correct procedures for the evaluation of risky cash outflows. *Journal of Financial and Quantitative Analysis*. 1982;17(2):287-300. DOI: 10.2307/2330851
16. Brigham E.F., Gapenski L. Intermediate financial management. Chicago, IL: Dryden Press; 1987. 816 p.
17. Copeland T.E., Weston J.F. Financial theory and corporate policy. Reading, MA: Addison-Wesley; 1988. 946 p.
18. Brown S.P.A. A note on environmental risk and the rate of discount. *Journal of Environmental Economics and Management*. 1983;10(3):282-286. DOI: 10.1016/0095-0696(83)90034-7
19. Prince R. A note on environmental risk and the rate of discount: Comment. *Journal of Environmental Economics and Management*. 1985;12(2):179-180. DOI: 10.1016/0095-0696(85)90027-0
20. Gallagher T., Miao H., Ryan P. Implied risk adjusted discount rates and certainty equivalence in capital budgeting. *Global Journal of Accounting and Finance*. 2017;1(2):25-30.
21. Brealey R.A., Myers S. Principles of corporate finance. 7th ed. New York: McGraw-Hill Book Co.; 2003. 1071 p. (Russ. ed.: Brealey R., Myers S. Printsipy korporativnykh finansov. Moscow: Olymp-Business; 2008. 1120 p.).
22. Kulakov N., Kulakova A. Evaluation of nonconventional projects. *The Engineering Economist*. 2013;58(2):137-148. DOI: 10.1080/0013791X.2012.763079
23. Blaset Kastro A., Kulakov N. Alternative evaluation methods for non-conventional investment projects. *Korporativnye finansy = Journal of Corporate Finance Research*. 2017;11(1):111-128. (in Russ). DOI: 10.17323/j.jcfr.2073-0438.11.1.2017.111-128
24. Beedles W.L. Evaluating negative benefits. *Journal of Financial and Quantitative Analysis*. 1978;13(1):173-176. DOI: 10.2307/2330532
25. Gitman L.J., Zutter C.J. Principles of managerial finance. 13th ed. Harlow: Pearson Education Ltd.; 2015. 850 p.
26. Keown A.J., Martin J.D., Petty J.W. Foundations of finance. 10th ed. Boston, MA: Pearson Education, Inc.; 2016. 610 p.

27. Heaton H.B. On valuing negative cash flows related to contamination, asset removal, or functional obsolescence. *Journal of Property Tax Assessment and Administration*. 2005;2(4):33-41.

28. Korn G., Korn T. Mathematical handbook for scientists and engineers: Definitions, theorems, and formulas for reference and review. New York: McGraw-Hill Book Co.; 1968. 943 p. (Russ. ed.: Korn G., Korn T. Spravochnik po matematike dlya nauchnykh rabotnikov i inzhenerov. Moscow: Nauka; 1984. 832 p.).

29. Robichek A.A., Myers S.C. Conceptual problems in the use of risk-adjusted discount rates. *The Journal of Finance*. 1966;21(4):727-730. DOI: 10.1111/j.1540-6261.1966.tb00277.x